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Bank Panics and the Endogeneity of Central Banking

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Bank Panics and the Endogeneity of Central Banking

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Abstract

Central banking is intimately related to liquidity provision to banks during times of crisis, the lender-of-last-resort function. This activity arose endogenously in certain banking systems. Depositors lack full information about the value of bank assets so that during macroeconomic downturns they monitor their banks by withdrawing in a banking panic. The likelihood of panics depends on the industrial organization of the banking system. Banking systems with many small, undiversified banks, are prone to panics and failures, unlike systems with a few big banks that are heavily branched and well diversified. Systems of many small banks are more efficient if the banks form coalitions during times of crisis. We provide conditions under which the industrial organization of banking leads to incentive compatible state contingent bank coalition formation. Such coalitions issue money that is a kind of deposit insurance and examine and supervise banks. Bank coalitions of small banks, however, cannot replicate the efficiency of a system of big banks.

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I. Introduction

The most important function of a central bank is to provide liquidity to the banking system in times of crisis. The classic work on central banking, *Lombard Street*, by Walter Bagehot, published in 1873, offered the advice that in times of panic the central bank (Bank of England) should lend freely and continue to pay out currency. At the time *Lombard Street* was published, there was no central bank in the U.S. and yet the private arrangement of banks in the U.S. clearinghouse system had already discovered Bagehot’s precepts and was acting on them. In this paper we argue that the lender-of-last-resort function of “central banking” arose endogenously through the formation of state contingent bank coalitions, such as clearinghouses, which provided liquidity during banking panics. Moreover, in the model we propose, banking panics are not irrational manifestations of multiple equilibria. Rather, they represent depositors monitoring their banks in banking systems composed of many small banks. Since panics do not happen in banking systems with large, heavily branched, well diversified, banks, this is consistent with the historical experience of countries internationally.

A thesis of this paper is that central banking (the lender-of-last-resort) emerges as a response to the banking system’s problems. The problem is the ability of depositors to monitor their banks. Monitoring banks corresponds to banking panics, and such panics may involve inefficiencies because banks may be mistakenly liquidated. Another thesis we develop is that banking panics are not a manifestation of an inherent problem with banks per se. On the contrary, the theory we develop views panics as a rational form of monitoring of banks by uninformed depositors. Briefly, depositors know the state of the macroeconomy, but not the idiosyncratic state of their own bank. The way to check on a bank is to ask the bank to convert its demand deposits into currency. But, banks as a whole cannot do this and then the banking system faces liquidation. Historically, and in the model we present, banks form coalitions that can turn illiquid loan portfolios into liquid claims that can convince depositors that the banks, as a group, are solvent, even if a depositor’s particular bank is not. This is the lender-of-last-resort function.

The model closely follows the U.S. experience with panics, not surprisingly since the U.S. is the leading example of the case of frequent banking panics. In the case of the U.S., liquidity provision by banks literally took the form of private money called “clearinghouse loan certificates.” These certificates were issued by bank coalitions and functioned as a form of deposit insurance from the point of view of depositors because they served to convert claims on a single bank into claims on the group of banks in the coalition. For depositors to accept these
certificates required that banks coinsure each other. Moreover, for this insurance system to work, there must be banking panics that impose externalities on the banks doing well, forcing them to subsidize other banks. We show how banking panics play a critical economic function enforcing the incentive compatibility of bank coalitions. In order for the bank coalition to form during times of panic, the banks had to agree to mutually monitor each other to enforce reserve and capital requirements. This monitoring is the historical origin of bank examination and supervision.

While bank coalitions in the U.S. were highly developed because of the industrial organization of the U.S. banking system, bank coalitions appear to be part of the banking histories of most countries, as we discuss below. Sometimes coalitions were formal arrangements; sometimes they were informal arrangements. Sometimes the coalitions were organized around a single dominant bank, such as the Suffolk Bank of pre-Civil War New England, J.P. Morgan in the 19th century, or the Bank of Montreal in Canada. Sometimes private banks formed a coalition with the government, as British banks often did with the Bank of England. Not all these coalitions issued private money directly to the public, but all had features of coinsurance that correspond to liquidity creation.

The extent to which coalitions were formed and were formalized is related to the frequency with which they were called upon to provide liquidity. This likelihood is related to some other important facts about banking history. In particular, historically, there is significant cross section variation in countries’ panic experiences. Historical studies have led to a consensus that the cross section variation in panic experience is due to variation in the industrial organization of the banking system. We review some of the evidence below. Such evidence is important because it suggests that panics are not an inherent feature of banking per se, as is commonly supposed and asserted theoretically. Our study is explicitly aimed at showing the importance of this industrial organization and its relation to the frequency of panics and bank failures.

The model we analyze is simple. There are two core assumptions. First, there is asymmetric information between banks, who are better informed, and their depositors. Second, banks may engage in moral hazard if their equity falls below a critical value. These are fairly standard assumptions. As one might expect, these assumptions lead depositors to sometimes want to withdraw their bank deposits. Withdrawals may be inefficient because the bank may, in fact, be quite well off, but depositors do not know this. We consider different organizational forms of the banking industry, systems with a few highly branched, well diversified, banks, as well as systems
with many small independent unit banks. Panics do not occur in all of these systems, though withdrawals do. This is because the need for monitoring by depositors varies depending on the nature of the banking system, corresponding to economic history.

There is a large existing literature on banking panics (See Gorton and Winton (2002) for a review of this literature). The dominant view among theorists emanates from Diamond and Dybvig (1983) who see banks as inherently unstable institutions prone to panics. Our view is different. It is closer to that of Calomiris and Kahn (1991) who also see panics as a monitoring device. But, the critical feature of their model is that depositors, who produce private information about the bank run, get into line first to withdraw. With first-come-first-served as the allocation rule, such information-producing depositors can cover the cost of their information production. Thus Calomiris and Kahn explain the first-come-first-served rule. But, this does not generate system wide panics, but rather runs on individual banks. We do not assume a first-come-first-served rule.

We show that a system of large, well-diversified, banks is more efficient than a system of many small, independent, unit banks. A bank coalition can improve the efficiency of the system of small banks, but it cannot achieve the allocation provided by the system of large banks. The government cannot improve upon either the coalition system or the system of large banks unless (i) the government is assumed to have much more power than private agents, e.g., it can seize bank assets; (ii) there are negative externalities that the banks or the coalition fail to internalize; or (iii) the government has access to resources outside the model. It is difficult to find an economic rationale for the government to be the lender-of-last-resort, though see Gorton and Huang (2001).

The paper proceeds as follows. In Section II we very briefly review some of the historical and cross-country evidence on the performance of banking systems and the history of panics. Our aim is to develop a set of stylized facts that a theory of panics and endogenous bank regulation should address. In Section III we present a simple model of a banking system that is then analyzed in subsequent sections. Our first step is to analyze two polar cases using the model. The first case is a banking system with small independent unit banks (Section IV) and the second is a system of large, well-diversified, branched banks (Section V). Neither of these systems literally represents reality, though they come close to the experiences of some countries. The U.S. historically has been a system of small independent unit banks and when private
clearinghouses were in existence, not all banks were members. The system of large branched banks, the other polar case, does resemble many of the world’s banking systems, such as Canada. But, even these systems occasionally have problems that necessitate coalition formation, as we briefly discuss below. In Section VI we consider the system with small independent unit banks that can form a coalition in the event of a banking panic. Section VII concludes.

II. An Overview of the Historical and Cross Country Evidence


The most important empirical regularity is that the industrial organization of the banking industry is a critical determinant of the propensity for an economy to experience panics. Banking panics are less likely to occur in banking systems in which there are a few relatively large, well-branched, and well-diversified banks. Surveying the international evidence, Calomiris (1993) cites industrial organization as the single most important factor explaining the incidence of panics. This is also the conclusion of Bordo (1986) who studies the experiences of six countries (U.S., U.K., Canada, Sweden, Germany, and France) over the period 1870 to 1933. Bordo (1985) concludes: “the United States experienced panics in a period when they were a historical curiosity in other countries” (p. 73). See also Grossman (1994). Studies of cross section variation within the United States lead to the same conclusion. In particular, states that allowed branching experienced lower failure rates. See Calomiris (1990, 1993), Bremer (1935) and White (1983, 1984). It is simply not the case panics are inherent to banking.

A comparison of the U.S. and Canadian banking experiences from the middle of the 19th century is a particularly instructive example of the importance of industrial organization in banking and its relation to central banking. Haubrich (1990), Bordo, Rockoff, and Redish (1994, 1995), and White (1984), among others, study the drastic contrast between these two systems. During the period 1870 to 1913, Canada had a branch banking system with about forty chartered banks, each extensively branched, while at the same time the United States had thousands of banks that could

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1 Some banks were too far away to be members. Rural banks and banks in smaller cities did not have formal clearinghouse arrangements.
not branch across state lines. The U.S. experienced panics, while Canada did not.\textsuperscript{2} There were high failure rates in the U.S. and low failure rates in Canada. Thirteen Canadian banks failed from 1868 to 1889, while during the same period hundreds of bank failed in the U.S. (see the Comptroller of the Currency (1920)). During the Great Depression, there were few bank failures in Canada, but the Canadian banking system did shrink by the about the same amount as in the U.S. (see White (1984)). Overall, the Canadian banking system survived the Great Depression with few effects, while in the U.S., which had enacted the Federal Reserve Act in 1914, the banking system collapsed. Canada’s central bank came into being in 1935, well after the Great Depression.

A second apparent regularity concerning banking panics is that there is an important business cycle and, possibly, seasonal component to the timing of panics. Panics come at or near business cycle peaks. The interpretation is not that panics caused downturns. There is not enough data to analyze that issue (at least, to date). Rather, the idea is that depositors received information forecasting a recession and withdrew in an anticipation of the recession, a time when bank failures were more likely. See Bordo (1986), Gorton (1988), Calomiris and Gorton (1991), and Donaldson (1992). While the relation of panics to the business cycle will be incorporated into the model below, the “regularity” is somewhat fragile as there are few observations of panics.\textsuperscript{3}

Associated with the likelihood of bank panics is the prevalence of private arrangements among banks. In the U.S., for example, where panics were not infrequent, there was the development of the private clearinghouse system. The U.S. clearinghouse system developed over the course of the 19\textsuperscript{th} century (see Andrew (1908b), Sprague (1910), Cannon (1910), Timberlake (1984), Gorton (1984, 1985), Gorton and Mullineaux (1987), Moen and Tallman (2000), and Wicker (2000), among others). For purposes here the main point concerns the method clearinghouses developed to turn illiquid loan portfolios into money, private money that could be handed out to depositors in exchange for their demand deposits during times of panic. During a banking panic member banks were allowed to apply to a clearinghouse committee, submitting assets as collateral in exchange for certificates. If the committee approved the assets, then certificates

\textsuperscript{2} Calomiris and Gorton (1991) identify six panics in the United States prior to 1865, seven during the National Banking Era.

\textsuperscript{3} Some have argued that there is also a seasonal factor in panics. The seasonal factor in the timing of panics is noted by Andrew (1907), Kemmerer (1910), Miron (1986), Donaldson (1992), and Calomiris and Gorton (1991), among others. But, Wicker (2000), for example, disputes the evidence. The seasonal factor seems less clear than the business cycle component, but could easily be incorporated.
would be issued only up to a percentage of the face value of the assets. During the Panics of 1873, 1893, and 1907 the clearinghouse loan certificate process were issued directly to the banks’ depositors, in exchange for demand deposits, in denominations corresponding to currency. If the depositors would accept the certificates as money, then the banks’ illiquid loan portfolios would be directly monetized. The loan certificates were the joint liability of the clearinghouse, not the individual bank. In this way, a depositor who was fearful that his particular bank might fail was able to insure against this event by trading his claim on the individual bank for a claim on the portfolio of banks in the clearinghouse. This was the origin of deposit insurance.

Bank coalitions are also not unique to the United States. There are many examples of bank coalitions forming on occasion in other countries as well (see Cannon (1908, 1910) for information on the clearinghouses of England, Canada, and Japan). We mention a few examples. According to Bordo and Redish (1987) “the Bank of Montreal (founded in 1817) emerged very early as the government’s bank performing many central bank functions. The pattern of the Bank of Montreal (and earlier precursors like the Suffolk Bank in the U.S.) in which the bank coalition is centered on one large bank, is quite common. Another common feature is the cooperation of a (perhaps, informal) coalition of banks with the government to rescue a bank in trouble or stem a panic. For example, major Canadian banks joined with the Canadian government to rescue the Canadian Commercial Bank in March 1985. (See Jayanti, Whyte, and Do (1993).) Similarly, in Germany the Bankhaus Herstatt was closed June 26, 1974. There was no statutory deposit insurance scheme in Germany, but the West German Federal Association of banks used $7.8 million in insurance to cover the losses. Germany is a developed capitalist country where deposit insurance is completely private, being provided by coalitions of private banks that developed following the Herstatt crisis of 1974. See Beck (no date).

The model below is developed to be consistent with these stylized facts.

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4 The amount of private money issued during times of panic was substantial. During the Panic of 1893 about $100 million of clearinghouse hand-to-hand money was issued (2.5 percent of the money stock). During the Panic of 1907, about $500 million was issued (4.5 percent of the money stock). See Gorton (1985).

5 The clearinghouse system was not the only private central bank-like institution in U.S. history. Before the U.S. Civil War, coincident with the beginnings of the clearinghouse system, the Suffolk Bank of Massachusetts was the focal point of a clearing system and acted as a lender-of-last-resort during the Panic of 1837. See Mullineaux (1987), Calomiris and Kahn (1996), Rolnick, Smith, and Weber (1998a, 1998b), and Whitney (1878).
III. The Model

There are three dates, 0, 1, and 2 in the model economy and two types of agents: consumers/depositors and bankers. Bankers are unique in having the ability to locate risky investment opportunities. Also, only banks can store endowments (i.e., provide the service of safekeeping).

There is a continuum of bankers. Each banker has capital $\beta$ and a measure one of potential depositors. Each bank has access to a riskless storage technology and to a risky investment technology. The fraction of the portfolio invested in the riskless storage technology is $\alpha$; this investment will be referred to as reserves. The remaining fraction $1-\alpha+\beta$ is invested in the risky technology. Investments in the risky projects have to be made at date 0, and the returns are realized at date 2. The date 2 return to a risky project depends on the state of the economy, which is a random variable, realized at date 1. The return to a unit (of endowment good) invested in the risky project is $\bar{\pi} + \tilde{r}$, that is, there is a systematic component, $\bar{\pi}$, and an idiosyncratic component, $\tilde{r}$, to the return. So, the state of the macroeconomy is indicated by $\bar{\pi}$, while the bank’s individual prospects are indicated by $\tilde{r}$. We assume that $\bar{\pi}$ is uniformly distributed in the interval $[\pi_L, \pi_H]$. For future reference, the probability density function of $\bar{\pi}$ will be referred to as $A$, where $A = \frac{1}{\pi_H - \pi_L}$. The idiosyncratic return for a risky project, $\tilde{r}$, is uniformly distributed in the interval $[0, 2M]$. For future reference, the mean of $\tilde{r}$ is denoted by $M$, i.e., $M = \frac{0 + 2M}{2}$.

At date 1, information about the date 2 return is realized, but there is asymmetric information between bankers and depositors. Depositors observe the realized state of the macroeconomy ($\pi$), but they do not observe the realized state of their bank’s idiosyncratic return ($r$). Each banker knows his own bank’s state ($r$), and observes the realizations of other banks’ idiosyncratic shocks at date 1. Idiosyncratic shock realizations at date 1 are not verifiable among banks, but realized cash flows at date 2 are verifiable. So, to be clear, banks cannot write contracts with other banks contingent on idiosyncratic shocks at date 1. At date 0, we assume that banks’ choice of reserve level $\alpha$ and the level of bank capital $\beta$, are observable and verifiable.

There is a moral hazard problem in that bankers have an opportunity to engage in fraud at date 1. Fraud is socially wasteful. If a banker engages in fraud, he gets a proportion $f$ of the return (i.e. $f(\pi+r)$), where $f$ is between 0 and 1. The remaining amount, $(1-f)(\pi+r)$, is wasted and depositors
receive nothing. Projects can be liquidated at date 1, yielding a constant return of \( Q \), regardless of the state of the project.

Depositors have a subsistence level of 1. Their utility function is:

\[
u(c_1, c_2) = \begin{cases} 
  c_0 + c_1(1 + \varepsilon_1) + c_2(1 + \varepsilon_2) & \text{if } c_0 + c_1 + c_2 \geq 1 \\
  -\infty & \text{if } c_0 + c_1 + c_2 < 1
\end{cases}
\]

where \( c_0, c_1 \) and \( c_2 \) are consumptions at date 0, 1 and 2 respectively. \( \varepsilon_1 \) and \( \varepsilon_2 \) represent depositors’ preference for later consumption. We assume \( \varepsilon_2 > \varepsilon_1 > 0 \), and they are both very small such that they can be ignored in the following analysis. Depositors' utility function implies that they will always wait until date 2 to withdraw if they believe their deposits are safe. However, they will withdraw at date 1 if they anticipate that there is any chance that their bankers are going to engage in fraud. Depositors deposit in a single bank.

Finally, bankers are risk neutral and they get the entire surplus from investment.

We assume the following:

**Assumption 1.** \( \frac{\pi_L + \pi_H}{2} + M > 1 > Q \). This assumption says that, ex ante, a risky project is more efficient than riskless storage, if there is no liquidation or fraud. However, if liquidation or fraud happens, then a risky project is dominated by investment in riskless storage.

**Assumption 2.** \( (1+\beta)(1-f)(\pi_L+M) < 1 \). This assumption assures that there is a potential moral hazard problem. Suppose a banker invests all of his assets in the risky project, and the economy turns out to be in the worst possible state (\( \pi_L \)) at date 1. Consider the banker with the mean return \( \pi_L+M \). If the banker engages in fraud, he will receive \( f(1+\beta)(\pi_L+M) \). If he does not engage in fraud, his payoff will be \( (1+\beta)(\pi_L+M) - 1 \). The assumption that \( (1+\beta)(1-f)(\pi_L+M) < 1 \) ensures that the banker has an incentive to engage in fraud.

**Assumption 3.** \( \pi_L > Q > f(\pi_H+2M) \). In words, there is a dead weight loss if liquidation or fraud occurs. If fraud does not occur, then the value of a risky project is greater than the liquidation value, \( Q \), even if the project is in the lowest possible state. If fraud occurs, then the value of a risky project is less than the liquidation value even if the project is in the highest possible state.
Assumption 4. A risky project is indivisible when liquidation occurs. Although at date 0, a banker can choose how much to invest in a risky project, at date 1 all the assets in a risky project must be liquidated if liquidation occurs.

Assumption 5. \((1+\beta)Q>1\). That is, if depositors withdraw from their bank at date 1, then their deposit contract can always be honored.

Under these assumptions, a banker may have an incentive to engage in moral hazard in certain states of the world. Depositors, however, are rational. If they anticipate that the bankers are going to engage in fraud, they withdraw their deposits to prevent it. Bankers can commit to not engage in moral hazard by holding reserves. The higher the level of reserves, the lower the probability that a bank run occurs. However, ex post, if the state of the economy is good at date 1, then it would have been better to have invested in risky projects. The bankers' task at date 0 is to choose an optimal reserve level, \(\alpha\) (the fraction of bank assets held in the riskless storage technology). This is the only choice variable. The optimal reserve choice depends on whether bank branching is allowed and on the interaction between the bankers. We interpret branching restrictions and different interactions between the bankers as different banking systems. Below, we will solve the bankers' optimization problem under the different organizations of the banking industry, examining the reserve level, banking stability, and social welfare under each system.

The essential ingredients of the model are the information asymmetry and the moral hazard problem. In particular, at date 1 each banker has private information about the idiosyncratic shock to his own bank. Based on this information the banker may have an incentive to engage in fraud. Depositors want to monitor banks to prevent this from happening, but have only one tool at their disposal: they can withdraw from the bank. The assumed information structure is meant to capture an essential feature of banking, namely, that the value of bank assets is opaque. See, e.g., Morgan (2000).

The moral hazard problem in this model is fraud. This is realistic as fraud has historically been the most common reason for bank failure. See Comptroller of the Currency (1920, 1988a, 1988b), Benston and Kaufman (1986). The Comptroller of the Currency (1873), reporting on the Panic of 1873, wrote that all the bank failures during the panic were due to “the criminal mismanagement of their officers or to the neglect or violation of the national-bank act on the part of their directors” (p. xxxv). A century later, the Comptroller of the Currency (1988b) reported that:
The study found insider abuse in many of the failed and rehabilitated banks during their decline. Insider abuse—e.g., self-dealing, undue dependence on the bank for income or services by a board member or shareholder, inappropriate transactions with affiliates, or unauthorized transactions by management—was a significant factor leading to failure in 35 percent of the failed banks. About a quarter of the banks with significant insider abuse also had significant problems involving material fraud. (p. 9).

For purposes of the model, it is important that there be a moral hazard problem, but it is not essential that the problem be fraud. Any one of a number of moral hazard problems would suffice. Fraud, however, is a realistic and significant problem.

There are two functions of banks in the model economy. First, banks are unique in being able to identify risky investment opportunities; consumers/depositors cannot find these opportunities. Second, banks can provide a claim, a demand deposit, which is consistent with the subsistence requirement of consumers. Because of their utility functions, consumers need to be assured that their claim will be worth 1 unit and banks can satisfy this need. Implicitly, individual banks can diversify to this extent. Gorton and Pennacchi (1990) show that uninformed consumers/traders with uncertain consumption demands prefer to transfer wealth intertemporally with claims that are riskless. Riskless claims are not subject to trading losses to informed traders if consumers need to consume at date 1 rather than date 2. The realization of date 1 consumption needs forces these uninformed consumers to sell their claims on a market where they lose money to informed agents. A better arrangement for these consumers could be claims on a diversified bank that are always worth 1 unit (i.e., so that there is no private information that informed traders could learn). We do not explicitly incorporate all this here. Rather, in the model here the structure of preferences dictates the type of claim that banks will offer depositors: the bank must offer the right to withdraw deposits at face value at date 1, i.e., a demand deposit contract.

We now turn to examining the functioning of the banking system when it is organized in different ways. We consider three basic forms of organization, two polar cases and one intermediate case. The first case is a system of many small independent unit banks. The next is a system of large, well-diversified banks, and the last is a system of small unit banks that can form a coalition in certain states of the world.

IV. The System of Independent Unit Banks

The first banking system we examine is one in which there are many small, independent unit banks. That is, implicitly the banks are small so they are undiversified. This is because they have
no branches and they do not interact with each other ex ante or ex post (they are independent). This system characterizes those periods of U.S. history, for example, where banks were not allowed to branch and where they did not form explicit or implicit coalitions. We will call this banking system the “Unit Bank” system.

Unit Banks are “small” in the following sense:

**Assumption 6.** A banker in charge of a Unit Bank can only manage one risky project. A banker cannot diversify the risk by dividing his asset portfolio into many risky projects.

Implicitly, we imagine that banks are spatially separated so that risky projects have the idiosyncratic risk of the individual bank’s location. The assumption also implies that at date 1, the project of a banker cannot be transferred to another bank. A project becomes worthless when taken over by another bank/banker. In other words, a project involves a relationship specific investment that cannot be transferred.

We solve the bankers’ optimization problem by backward induction. First, given a Unit Bank's choice of reserve level, \( \alpha \), we characterize the states in which bankers will have incentives to engage in moral hazard and, hence, depositors will withdraw their deposits. Second, we will calculate the bankers’ optimal choice of reserve level, \( \alpha \), at date 0.

At date 1, depositors receive the signal about the state of the macroeconomy, \( \pi \); they expect that the return to their bank's risky project is \( \pi + E(r) \). Based on the state of the macroeconomy they infer whether their banker has an incentive to engage in fraud.

Depositors do not observe the realization of their bank’s idiosyncratic shock, \( r \). Because of their utility functions they do not care about the likelihood of their bank engaging in fraud, but only in whether there is any chance of this occurring. They, therefore, assume that \( r=0 \) and check whether their banker has an incentive to engage in fraud. This leads to:

**Lemma 1:** At date 1, given a banker’s reserve level \( \alpha \) and the realized state of the economy \( \pi \), if
\[
\pi < \frac{1 - \alpha}{(1-f)(1+\beta-\alpha)},
\]
then there is a positive probability that bankers will engage in fraud. If
\[
\pi \geq \frac{1 - \alpha}{(1-f)(1+\beta-\alpha)},
\]
then no banker has an incentive to engage in fraud.
**Proof:** Suppose the banker has reserves of $\alpha$ and the realized idiosyncratic shock is $r=0$. The realized state of the macroeconomy is $\pi$. If the banker does not engage in fraud, his payoff will be $\pi(1+\beta-\alpha)+\alpha-1$. If he engages in fraud his payoff will be $\pi f(1+\beta-\alpha)$, since he cannot steal anything from the reserves. If $\pi f(1+\beta-\alpha)>\pi(1+\beta-\alpha)+\alpha-1$, or $\pi<\frac{1-\alpha}{(1-f)(1+\beta-\alpha)}$, the banker engages in fraud. Otherwise he has no incentive to engage in fraud. //

Lemma 1 illustrates the viewpoint of depositors who do not know $r$, the realization of the idiosyncratic component of the return. They observe the state of the macroeconomy, $\pi$, and can calculate whether, given that state, there is a chance that bankers will engage in moral hazard. Because their utility functions are kinked and they will get minus infinity if consumption is less than one, depositors do not care about the likelihood of moral hazard occurring, but rather whether there is any chance of moral hazard occurring.

If depositors find that there is a chance that bankers will engage in fraud (i.e. $\pi<\frac{1-\alpha}{(1-f)(1+\beta-\alpha)}$), then they withdraw all their savings. Since all the depositors receive the same macroeconomic information and all the banks are, from their viewpoint, homogeneous, if one bank suffers from a run, there are runs on all the other banks. Therefore, a panic occurs.

**Lemma 2:** At date 1, given the banker's reserve $\alpha$ and the realized state of the economy $\pi$, if $\pi<\frac{1-\alpha}{(1-f)(1+\beta-\alpha)}$, then a banking panic occurs.

Note that the panic is defined by two characteristics of date 1 actions. First, depositors cannot distinguish between banks because of the lack of bank-specific information and so, if they choose to withdraw at date 1, they withdraw from all banks. Second, this causes all the banks to be liquidated, which in this model we can think of as failure since the banks go out of existence prematurely. In terms of the model, we emphasize this with the following definition.

**Definition:** A **banking panic** is a date 1 event in which depositors at all banks seek to withdraw their deposits and banks cannot honor these demands, resulting in a suspension of convertibility and the liquidation of at least some banks.

Note, for future reference, that if all depositors seek to withdraw their deposits at date 1, but their requests can be honored without suspension or liquidations of banks, then we do not deem this event to be a banking panic.
At date 0, anticipating what will happen in different states of the world at date 1, bankers choose the optimal reserve level to maximize their expected payoff. One the one hand, bankers want to maximize investment in the risky projects because this is more profitable, but on the other hand, they want to avoid being prematurely liquidated in a banking panic at date 1. According to Lemma 1 and Lemma 2, if bankers hold reserves such that
\[
\alpha \geq \alpha_{\text{max}} = \frac{1 - (1 + \beta)(1 - f)\pi_L}{1 - (1 - f)\pi_L},
\]
then they have no incentive to engage in the moral hazard, even if the economy is in the lowest state. Therefore, bankers solve the following optimization problem at date 0:

\[
\begin{align*}
\text{Max} & \quad \alpha \int \pi^r [\alpha + (1 + \beta - \alpha)(\pi - 1)]dF(\pi) + \int \pi^u [\alpha + (1 + \beta - \alpha)(\pi + M) - 1]dF(\pi) \\
\text{s.t.} & \quad \pi^r = \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)} \\
& \quad \alpha \in [0, \alpha_{\text{max}}]
\end{align*}
\]

The solution to the bankers’ date 0 problem is characterized in the following proposition.

**Proposition 1:** There exists a unique optimal reserve level \( \alpha \in [0, \alpha_{\text{max}}] \) that solves the bankers' optimization problem. Define: \( C^U = 1 - (\pi_L + \pi_H + M) + A\left[\frac{1}{2}(1 - f)^2 - (\frac{1}{1 - f} - \pi_L)(Q - M) - \frac{1}{2}\pi_L^2\right]. \)

If \( C^U \leq \frac{A\beta^2}{2(1 - f)^2(1 + \beta)^2} \), then the optimal \( \alpha = \alpha^U = 0 \);

If \( C^U \geq \frac{A\beta^2}{2(1 - f)^2(1 + \beta - \alpha_{\text{max}})^2} \), then the optimal \( \alpha = \alpha^U = \alpha_{\text{max}} \);

If \( \frac{A\beta^2}{2(1 - f)^2(1 + \beta)^2} < C^U \leq \frac{A\beta^2}{2(1 - f)^2(1 + \beta - \alpha_{\text{max}})^2} \), then there is an interior solution

\[
\alpha^U = 1 + \beta - \frac{\beta}{(1 - f)^2} \sqrt{\frac{A}{2C^U}}. \quad \text{Panic occurs whenever} \quad \pi \leq \pi^U = \frac{1 - \alpha^U}{(1 - f)(1 + \beta - \alpha^U)}.
\]

**Proof:** See Appendix.
The following corollary is clear from Proposition 1.

**Corollary 1:** The optimal $\alpha^U$ is decreasing in bankers' capital $\beta$.

The purpose of a panic is to monitor the bankers, to prevent them from engaging in fraud. The panic is not irrational; it is not motivated by externalities due to actions of other depositors when there is a sequential service constraint. Rather, the panic is related to the macroeconomy, which may create incentives for bankers to engage in moral hazard. The fear of not being able to satisfy subsistence should the banker engage in moral hazard, a kind of extreme risk aversion, causes the depositors’ withdrawals. However, not all bankers will engage in moral hazard. The problem is that depositors do not know which bankers have high idiosyncratic shock realizations and which have low idiosyncratic shock realizations. Depositors liquidate all banks. Because of this possibility, bankers hold high reserve levels, but this is inefficient.

V. The Big Bank System

At the other extreme from a banking system composed of many independent unit banks is a system where banks are large and heavily branched. We call this the “Big Bank” system. Most banking systems in the world are closer to this system than to the system of independent unit banks, discussed above.

**Definition:** A Big Bank is a bank with a portfolio of assets that has a realized return of $\pi + M$ at date 1.

So, a Big Bank’s return is the systematic return plus the diversified idiosyncratic mean return, $M$. This is the essential point, namely, that the idiosyncratic risk is diversified away, implicitly by virtue of the bank’s size via branching. Consequently, at date 1, when the state of the economy is revealed, the return to a Big Bank's risky projects is also known. The state of macroeconomy is sufficient information for assessing the state of a Big Bank. As a result, depositors know for sure whether a Big Bank is going to engage in moral hazard or not. If they anticipate that their Big Bank will engage in fraud, they run on the Big Bank. Otherwise they wait until date 2 to withdraw.

**Assumption 7:** Liquidation and fraud can occur at the project level.
Big Banks invest in a portfolio of projects, implicitly each being run by a branch manager. The Big Bank makes decisions about this portfolio of projects and, in particular, has the flexibility to liquidate individual projects and to engage in fraud at the level of these projects.

We solve the representative Big Bank’s problem in two steps. First, we begin by treating the bank by analogy with the system of independent unit banks. That is, we assume that a measure of such small banks can get together at date 0 and form a Big Bank. We determine when such a bank would face a bank run at date 1 and then determine the optimal reserve level chosen by this bank at date 0. We will draw some conclusions at this point. Then the second step of the analysis recognizes that a Big Bank can alter its portfolio at date 1, liquidating some projects while letting the other projects continue. So, the second step of the analysis takes this into account.

We proceed as above and solve the Big Bank's optimization problem using backward induction. First, given the Big Bank's choice of reserve level, we characterize the states in which a bank run occurs. Comparing the banker’s incentives, as above, leads to:

**Lemma 3:** At date 1, given the Big Bank's reserve level, \( \alpha \), and the realized state of the economy \( \pi \), if \( \pi + M < \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} \), the Big Bank will engage in fraud and therefore depositors will run on the Big Bank. If \( \pi + M \geq \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} \), the Big Bank has no incentive to engage in fraud and depositors will not run the bank.

At date 0, anticipating what will happen in different states of the world at date 1, a Big Bank chooses its optimal reserve level to maximize its expected payoff. If a Big Bank holds reserves such that \( \alpha \geq \alpha_{\max} = \frac{1-(1+\beta)(1-f)(\pi_L + M)}{1-(1-f)(\pi_L + M)} \), then it has no incentive to engage in moral hazard even if the economy is in the lowest state. A Big Bank solves the following optimization problem at date 0:

\[
\text{Max } \alpha \int_{\bar{\pi}} \left[ \alpha + (1 + \beta - \alpha)(\pi - 1)dF(\bar{\pi}) + \int_{\pi^*} \left[ \alpha + (1 + \beta - \alpha)(\pi + M) - 1 \right]dF(\bar{\pi}) \right]
\]

s.t. \( \pi^* + M = \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} \)

\( \alpha \in [0, \alpha_{\max}] \)
The solution is given in:

**Proposition 2:** There is a unique optimal \( \alpha \in [0, \alpha_{\text{max}}^B] \) that solves the Big Banks' optimization problem. Define \( C^B = 1 - \left( \frac{\pi_L + \pi_H}{2} + M \right) + A \left[ \frac{1}{1-f} - \left( \frac{1}{1-f} - M - \pi_L \right)(Q-M) - \frac{1}{2} \pi_2^L \right] \).

If \( C^B \geq \frac{A \beta^2}{2(1-f)^2(1+\beta)^2} \), then the optimal \( \alpha \) is \( \alpha^B = \alpha_{\text{max}}^B \).

If \( C^B \lesssim \frac{A \beta^2}{2(1-f)^2(1+\beta)^2} \), then the optimal \( \alpha \) is \( \alpha^B = \alpha_{\text{max}}^B \).

If \( \frac{A \beta^2}{2(1-f)^2(1+\beta)^2} < C^B < \frac{A \beta^2}{2(1-f)^2(1+\beta-\alpha_{\text{max}}^B)^2} \), then there is an interior solution, namely, \( \alpha^B = 1 + \frac{\beta}{1-f} \left[ \sqrt{A} \right] \). A bank run occurs when \( \pi \leq \pi^B = \frac{1-\alpha^B}{(1-f)(1+\beta-\alpha^B)} - M \).

The optimal \( \alpha^B \) is decreasing in bankers' capital \( \beta \).

**Proof:** See Appendix.

As mentioned at the outset of this section, Big Banks have another tool at their disposal that has not been taken into account yet. Big Banks have the flexibility to partially liquidate their portfolios at date 1. In fact, a Big Bank only needs to liquidate some of the risky projects when a bank run occurs. Recall that liquidation (and fraud) can occur at the project level. In order to deal with depositors’ withdrawals at date 1, a Big Bank only needs to liquidate a fraction \( x \) of the risky projects such that \( \alpha + (1+\beta-\alpha)xQ \) is equal to 1. Actually, however, a Big Bank can do even better if it can commit to not engage in fraud by liquidating some of the projects and holding the proceeds as additional reserves. Although the risky projects have idiosyncratic returns, they have the same liquidation value \( Q \). Suppose the Big Bank is to liquidate a fraction \( x \) of the risky projects. It should liquidate optimally, as follows. It will liquidate those projects that have realized idiosyncratic returns, \( r \), in the interval \( [0, x2M] \). The remaining \( (1-x) \) fraction of projects has realized idiosyncratic returns \( r \) in the complementary interval: \( [x2M, 2M] \). The average return on the remaining, i.e., nonliquidated, \( (1-x) \) fraction of projects is

\[ \pi + \frac{x2M + 2M}{2} = \pi + (1+x)M. \]

If the Big Bank allows the remaining projects to continue without engaging in fraud, its payoff is \( \alpha + (1+\beta-\alpha)xQ + (1+\beta-\alpha)(1-x)(\pi + (1+ x)M) - 1 \). If the
Big Bank engages in fraud on the remaining projects, its payoff will be \( f(1+\beta-\alpha)(1-x)(\pi + (1+x)M) \). Therefore, the Big Bank has to liquidate a fraction \( x \) of the risky projects such that: 
\[
\alpha + (1+\beta-\alpha)xQ + (1+\beta-\alpha)(1-x)(\pi + (1+x)M) - 1 \geq f(1+\beta-\alpha)(1-x)(\pi + (1+x)M)
\]
The optimal \( x \) is the solution to the following problem:

\[
\begin{align*}
\text{Max}_x & \quad \alpha + (1+\beta-\alpha)xQ + (1+\beta-\alpha)(1-x)(\pi + (1+x)M) - 1 \\
\text{s.t.} & \quad (1+\beta-\alpha)(1-x)(\pi + (1+x)M) - 1 \geq f(1+\beta-\alpha)(1-x)(\pi + (1+x)M) \\
x & \in [0,1].
\end{align*}
\]

The solution is given by:

**Lemma 4:** There is a unique \( x \in [0,1] \) that solves the above problem. The unique solution is:

\[
x = \frac{Q - (1-f)\pi - \sqrt{(Q - (1-f)\pi)^2 - 4M(1-f)(1-\alpha) + 4M(1-f)^2(\pi + M)}}{2M(1-f)}.
\]

Moreover, \( x \) is decreasing in \( \alpha, \beta \) and \( \pi \).

**Proof:** See Appendix.

Now the Big Bank’s problem at date 0 can be written as:

\[
\begin{align*}
\text{Max}_\alpha & \quad \int_{\pi_L}^{\pi_R} [\alpha + (1+\beta-\alpha)xQ + [(1+\beta-\alpha)(1-x)(\pi + (1+x)M) - 1]]dF(\pi) + \\
& \int_{\pi_H}^{\pi_R} [\alpha + (1+\beta-\alpha)(\pi + M) - 1]dF(\pi) \\
\text{s.t.} & \quad \pi_R + M = \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} \\
& \quad Q - (1-f)\pi - \sqrt{(Q - (1-f)\pi)^2 - 4M(1-f)(1-\alpha) + 4M(1-f)^2(\pi + M)} \\
& \quad 2M(1-f)
\end{align*}
\]

\( \alpha \in [0, \alpha_{\text{max}}] \)
Proposition 3: The above objective function is strictly concave in $\alpha$. There is a unique optimal reserve level, $\alpha \in [0, \alpha_{max}^B]$, that solves the big bank's optimization problem.

Proof: See Appendix.

To emphasize, note that in the Big Bank system banks may experience withdrawals at date 1, but they do not fail, i.e., they are not liquidated. In the model there is no difference between the bank liquidating projects and holding the proceeds as reserves and withdrawals. In other words, the Big Bank system can be viewed as experiencing deposit withdrawals, but there are no bank runs or failures. The Big Bank system does not experience banking panics. The independent unit banks have bank runs, and failures, because each unit bank’s project is indivisible when liquidation occurs.

There is no closed form solution for the representative Big Bank’s choice of $\alpha$ at date 0 when it has the flexibility to choose which projects to liquidate at date 1. But, it is clear that the system with flexibility is even more efficient than the Big Bank system without this flexibility. (A portfolio of options is more valuable than an option on a portfolio.)

In broad outlines, the distinction between the Big Bank system and the system of small independent Unit Banks corresponds to the difference between the Canadian and U.S. systems. As mentioned above, the Canadian system generally displayed fewer failures and no panics. In addition, as the table below makes clear, Canadian banks held fewer reserves (in the form of securities) and, correspondingly, they made more loans per asset dollar.
### Table: Bank Balance Sheet Items for Canada and the U.S., 1870-1919

<table>
<thead>
<tr>
<th></th>
<th>1870-79</th>
<th>1880-89</th>
<th>1890-99</th>
<th>1900-09</th>
<th>1910-19</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Canada</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loan/Assets</td>
<td>0.717</td>
<td>0.706</td>
<td>0.696</td>
<td>0.722</td>
<td>0.640</td>
</tr>
<tr>
<td>Securities/Assets</td>
<td>0.013</td>
<td>0.021</td>
<td>0.071</td>
<td>0.087</td>
<td>0.110</td>
</tr>
<tr>
<td>Debt/Equity</td>
<td>1.458</td>
<td>1.914</td>
<td>2.796</td>
<td>4.232</td>
<td>6.876</td>
</tr>
<tr>
<td><strong>United States</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loan/Assets</td>
<td>0.487</td>
<td>0.563</td>
<td>0.589</td>
<td>0.546</td>
<td>0.567</td>
</tr>
<tr>
<td>Securities/Assets</td>
<td>0.253</td>
<td>0.169</td>
<td>0.117</td>
<td>0.164</td>
<td>0.168</td>
</tr>
<tr>
<td>Debt/Equity</td>
<td>1.826</td>
<td>2.334</td>
<td>2.620</td>
<td>4.184</td>
<td>5.352</td>
</tr>
</tbody>
</table>

Source: Table 4 of Bordo, Redish, and Rockoff (1995) (based on U.S. Comptroller of the Currency, Annual Reports, and Curtis (1931)).

After 1920, the comparison is also stark. By 1920 the private clearinghouse system in the U.S. that functioned as a lender-of-last-resort was gone, having been replaced by the Federal Reserve System. In Canada, the bank merge movement, from 1900 to 1925, reduced the number of banks and resulted in a small number of banks with large branch networks. Prior to the merger movement, Canadian banks were branched, but there were many more banks. The post-merger movement banking system in Canada is clearly the Big Bank system. The comparison between the two systems during this period is the subject of Bordo, Rockoff, and Redish (1994), quoted above, who emphasize the fact that between 1920 and 1980 there was one bank failure in Canada, in contrast to hundreds and thousands in the U.S., particularly during the Great Depression. There were no banking panics in Canada, though the reduction in deposits during the Great Depression was of similar magnitude, as noted above.

### VI. Bank Coalitions

The above sections analyzed two polar case banking systems. Arguably, there are banking systems that resemble the Unit Banking system or the system of Big Banks, but as discussed above, historically bank coalitions have almost always been present in some form. The above two cases, the Unit Banking system and the Big Bank system, can be thought of as representative
benchmarks. In this section, we introduce the possibility of a bank coalition, i.e., a state contingent agreement between banks. The discussion of bank coalitions will follow the U.S. clearinghouse experience, briefly described above, but the argument is more general, as discussed below.

The basic idea for the coalition is as follows. The failure of individual small Unit Banks as a result of bank runs at date 1, despite holding high levels of reserves, can be improved upon if the small banks can replicate, at least partially, the performance of a Big Bank. Big Banks are diversified and so face different incentives to engage in moral hazard. In particular, Big Banks can liquidate part of their portfolio, in the face of withdrawals, and increase their reserves. For small banks to attempt to replicate the performance of a Big Bank, a certain level of diversification must be achieved. Then, at date 1, if there is a bank panic, coalition members can then credibly form a Big Bank by combining their assets and liabilities.

Credibility of the coalition is established by a signal of coalition solvency; the signal is the coalition’s act of issuing claims to depositors in exchange for individual bank deposits. These claims, the loan certificates, are supported by a sharing rule that combines assets and liabilities at date 1 and which provides incentives for the member banks with high idiosyncratic shock realizations to monitor member banks with low idiosyncratic shock realizations. “Monitoring” means preventing member banks from engaging in moral hazard, by liquidating these banks or subsidizing them. The internal workings of the coalition are not observable to depositors, so they will not accept the loan certificates unless they believe that the coalition’s behavior will, in fact, be as described above. In equilibrium depositors’ beliefs will be consistent with the behavior of the coalition. We now turn to providing the details.

A. The Setting With Bank Coalitions

Suppose that there are small independent Unit Banks at date 0. They are prohibited from forming a Big Bank. (For example, banks are prohibited from branching across state lines.) Without forming a Big Bank, however, these small Unit Banks can decide to form a coalition at date 0 and the coalition partially replicates the Big Bank in certain states of the world at date 1. The coalition will be a rule indicating that some banks are to be liquidated and a sharing rule for the remaining banks. At date 0, Unit Banks can get together to form a coalition and reach an agreement about their individual capital and reserve levels. Because the idiosyncratic shocks are not verifiable, and thus not contractible, the coalition has no power to force its members to comply with the rules and the member banks are free to quit at any time they want. The only
requirement to become a member of the coalition at date 0 is to hold the required reserve level (and capital level). At date 1 the depositors cannot observe whether the rules have been carried out or not. They can only observe whether the coalition liquidates some of the member banks and combines the assets and liabilities of the remaining member banks.

The sequence of events at date 1 begins with depositors observing the realized state of the macroeconomy and deciding whether to withdraw their deposits or not. Then the banks decide whether to trigger the operation of the coalition. (Subsequent subgames are explained below.) We define the coalition and the operation of the coalition as follows:

**Definition**: The bank coalition is an agreement between member banks at date 0 about the following issues:

(i) Bank reserve levels, $\alpha$, at date 0.

(ii) A date 1 state-contingent rule, $P(\alpha, \pi)$, indicating when the coalition is to operate ($P=1$) or not operate ($P=0$), in which case banks act as Unit Banks. (The contingency, in fact, will be a panic; this is shown below in Lemma 4.)

(iii) If the coalition is set into operation, then the coalition applies two rules: a liquidation rule and a debt transfer rule. The first rule, $L(\alpha, \pi, r)$, is a mapping from $[0, 2M]$ to \{1, 0\}, indicating whether a member bank (in state $\{\alpha, \pi\}$) with idiosyncratic shock $r$ is to be liquidated ($L=1$) or not liquidated ($L=0$). If the member is not liquidated, then the second rule applies. That rule, $D(\alpha, \pi, r)$, is a mapping from $[0, 2M]$ to $R^+$, indicating the liability reallocated to a member bank with idiosyncratic shock $r$.

If at date 1 the coalition is set into operation, then depositors observe that the coalition suspends convertibility in all banks. The coalition pools the liabilities of nonliquidated banks and issues loan certificates, which are debt claims to the coalition, backed by all the assets of all the member banks. Depositors also observe that the coalition liquidates some of the member banks.

Suppose that at date 1 the state of the world is such that the coalition operates. The issue of loan certificates and suspension of convertibility signal this to depositors. Then the coalition first applies the liquidation rule, ending the projects of some member banks. According to the second rule, all the non-liquidated members pool their liabilities together. A member bank with idiosyncratic shock $r$ is reallocated the liability $D(\alpha, \pi, r)$ by the coalition. Although $r$ is not
The liability \( D(\alpha, \pi, r) \) indicates how much of the coalition’s debt, the individual bank with idiosyncratic shock \( r \) is responsible for at date 2.

At date 2 the coalition has a budget constraint: \( \int D(\alpha, \pi, r) dF_{\text{NL}}(r) \geq 1 \), where \( F_{\text{NL}}(r) \) is the distribution function of the non-liquidated banks’ idiosyncratic shocks. If a member bank cannot honor its liability to the coalition, all the member banks that are solvent share the default amount. At date 2, if the coalition is solvent, all the solvent bankers receive what remains after they honor their individual liabilities, and honor their shared part of other members’ default liabilities. If the coalition is insolvent, none of the member banks can be solvent. Consequently only those bankers who engaged in fraud get the payoff from fraud.

If the coalition does not operate, each member bank acts as an independent bank and deals with its own depositors’ withdrawal demands.

The operation of the coalition is intended to achieve two goals. First, by liquidating some of the member banks the coalition tries to inform depositors that the non-liquidated banks are in relatively more sound states. This partially alleviates the panic caused by the asymmetric information between the banks and depositors. Second, by pooling the liabilities the coalition tries to convince depositors that incentives to engage in fraud can be removed by monitoring and coinsurance among the remaining banks. Whether the coalition is successful in achieving these goals will depend upon the beliefs of the depositors.

Before we study the equilibrium, we specify the following assumptions about the coalition.

**Assumption 8:** If a bank does not join the coalition at date 0, it cannot apply for membership at date 1.

This means that if a bank does not join the coalition at date 0, it acts as an independent Unit Bank. The purpose of this assumption is to save the complicated analysis of transactions between the coalition and non-members in different states at date 1.

**Assumption 9:** The identity of the date 0 coalition members, and the coalition itself, is observable at all dates.

\[ \text{Moreover, the coalition needs to prevent member banks revealing their } r \text{ by showing depositors their } D(\alpha, \pi, r). \text{ We can imagine that the coalition takes out a note } \text{“You owe the coalition } D(\alpha, \pi, r) \text{” and asks the banker for his signature. In this way, only the coalition holds the verifiable contracts, which specify all non-liquidated banks’ liabilities } D(\alpha, \pi, r). \]
This assumption has two implications. First, if a bank quits the coalition at date 1, then its depositors know that it is no longer a member of the coalition. Second, if a group of banks quit the original coalition and form another coalition, then depositors can distinguish the deviating coalition from the original coalition. (This assumption is for simplicity.)

**Assumption 10:** At date 1, bank coalition members learn the realizations of other members’ idiosyncratic shocks, r.

Bank coalitions were involved in clearing each other’s liabilities and this is one mechanism through which information about other members’ states of the world was acquired historically. In addition, the coalition can require that information be reported to the coalition. As discussed above, the clearinghouse required that certain information be reported and sent bank examiners to monitor members’ conditions.

**Assumption 11:** The coalition maximizes the total payoffs to its member banks.

For simplicity, we do not go into the details how decisions are made inside the coalition. We assume that the internal organization of the coalition is equivalent to assuming the existence of a coalition decision maker who is independent of any of the member banks and maximizes the total payoffs to all member banks.\(^7\)

**Assumption 12:** When a banker joins the coalition, property rights in his bank are maintained, that is, he cannot be forced to operate his bank in a certain way, nor can he be involuntarily separated from his bank’s assets.

Banks are private firms. Joining the coalition at date 0 does not change this. The intent of this assumption is to emphasize that while the coalition can liquidate member banks, it can only do so if the banker who owns the bank agrees. If a member bank is not liquidated, then the banks’ original owners must operate banks that are not liquidated. Under this assumption, non-liquidated good banks need to bribe/subsidize non-liquidated bad banks to keep them in the coalition, so that they do not engage in fraud. This restricts the coalition’s sharing rules because each banker has to be promised a payoff at least equal to the value to him from deviating and engaging in fraud. Otherwise the coalition would have more freedom to set the sharing rules because in order to make member banks accept the rules it only needs to threaten to drive them

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\(^7\) We take the basic features of the coalition, such as its objective function, as exogenous, and do not investigate how the coalition comes into being or whether other types of coalitions would be superior in some way. These are interesting questions, but we do not pursue them in here.
out of the coalition. Keep in mind that the project of a banker about to engage in fraud cannot be transferred to another bank since, by assumption, bankers can only manage a single project. In other words, we have assumed that a project becomes worthless when taken over by another bank/banker. A project’s value depends on a relationship specific investment that cannot be transferred.

B. Equilibrium with Bank Coalitions

The sequence of events begins at date 1 with depositors observing the state of the macroeconomy and deciding whether to withdraw or not. To simplify the exposition of the equilibrium we will start at this node of the game and then go on to the remaining subgames. This first decision of depositors is the decision to panic or not, and so deserves attention.

At date 1, each member bank of the coalition holds reserves of $\alpha$ when the state of the economy $\pi$ is realized. If $\pi \geq \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)}$, then even the bank with the lowest idiosyncratic shock (i.e., $r=0$) has no incentive to engage in fraud. Hence there is no need for depositors to run the banks. If $\pi < \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)}$, then some banks have an incentive to engage in fraud. We first show that without a bank panic, banks have no incentives to pool their liabilities.

**Lemma 4:** The coalition will not operate at date 1 if depositors do not panic.

**Proof:** See Appendix.

The lemma says that if there is no panic no bank coalition will operate. Banks will behave as Unit Banks. Because the rules of the coalition, adopted at date 0, are not binding, banks are free to deviate from those rules. They can, in principle, adopt any set of rules concerning transfers among members (as long as such rules satisfy the budget constraint for the coalition). The lemma says that any such set of transfers is dominated by banks acting as Unit Banks.

The banking panic creates an externality for banks that would not engage in the moral hazard problem, the “good” banks. If these good banks did not face the panic, they would have no incentive to monitor the banks that are going to engage in fraud, the “bad” banks. Because depositors cannot distinguish good banks from bad banks, all banks face the prospect of being liquidated. This creates the incentive for good banks to monitor bad banks.
Depositors anticipate that if they do not run the banks, the coalition will not do anything to prevent member banks from engaging in fraud. So, they run all banks if and only if
\[ \pi < \frac{1 - \alpha}{(1 - \gamma)(1 + \beta - \alpha)}. \] Once the depositors run the banks, the coalition has to operate to convince the depositors that it can remove the incentives to engage in fraud from some of its member banks and therefore there is no need to liquidate those banks.

We now turn to the subgames following the depositors’ panic decision. We define a coalition equilibrium in the face of a panic as follows:

**Definition:** When there is a bank panic, a coalition equilibrium consists of the strategies and beliefs of the various agents in the economy as follows:

1. To maximize the total payoff to all member banks, the coalition suspends convertibility and issues loan certificates. It then applies the liquidation rule by announcing which banks are liquidated, and applies the debt transfer rule by reallocating liabilities to the remaining banks.

2. Each member bank maximizes its payoff by choosing whether to accept the coalition’s rules or to quit the coalition.

3. Depositors observe: (i) the measure of banks liquidated by the coalition; (ii) which member banks quit the coalition; and (iii), which members remain in the coalition. Based on what they observe, they update their beliefs. On the equilibrium path, their beliefs are updated based on Bayes’ rule and the strategies taken by the coalition and its member banks. Off the equilibrium path, their beliefs have to be consistent with Bayes’ rule and the strategies of the coalition and its members. If their bank is liquidated by the coalition, then their deposits are paid off at date 1.\(^8\) If their bank is not liquidated, they need to decide whether to accept the loan certificates in exchange for demand deposits. If depositors believe their deposits are safe, they always withdraw at date 2. Otherwise they withdraw at date 1.

4. Non-liquidated banks decide whether to engage in fraud.

Based on the definition, the following proposition gives the coalition equilibrium in the face of a bank panic at date 1.

---

\(^8\) This is for simplicity. The alternative requires the liabilities of the liquidated banks to be honored by the coalition at date 2.
**Proposition 4:** Suppose that at date 1 \( \pi < \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)} \) and, consequently, depositors run the banks. Define \( x^*(\alpha, \pi) = \max\{0, \min\{1, \frac{1 - \alpha - \pi(1 - f)(1 + \beta - \alpha)}{M(1 - f)(1 + \beta - \alpha)} - 1\}\} \). In the coalition equilibrium:

1) The coalition operates; it issues loan certificates; \( P(\alpha, \pi) = 1 \). Then the coalition applies the liquidation rule, setting \( L(\alpha, \pi, r) = 1 \) (i.e., liquidation) for banks with idiosyncratic shocks \( r \in [0, x^*(\alpha, \pi)2M] \) and pays these bankers \( \alpha + (1 + \beta - \alpha)(\pi - r) \). For banks that are not liquidated, the complementary set, the coalition reallocates liabilities according to the members type, \( r \): \( D(\alpha, \pi, r) = \alpha + (1 - f)(1 + \beta - \alpha)(\pi + r) \);

2) No member bank quits the coalition;

3) Depositors believe that: (i) the coalition has liquidated the banks with idiosyncratic shocks in the interval: \([0, x^*(\alpha, \pi)2M]\); (ii) remaining member banks have idiosyncratic shocks distributed in the interval: \([x^*(\alpha, \pi)2M, 2M]\); and (iii), none of the non-liquidated member banks have incentives to engage in fraud. They liquidate any bank that quits the coalition and accept loan certificates issued by the coalition;

4) None of the member banks engage in fraud;

5) Off equilibrium path beliefs are as follows. If depositors observe a fraction \( y \) of banks quit the coalition, a fraction \( z \) of the banks remain non-liquidated in the coalition and a fraction \( 1 - y - z \) of banks are liquidated by the coalition, then they believe that the idiosyncratic shocks of those banks out of the coalition are distributed in the interval: \([0, y2M]\), and that the idiosyncratic shocks of those non-liquidated banks in the coalition are distributed in the interval: \([1 - y)2M, 2M]\). They liquidate all the banks out of the coalition, and accept the loan certificates if and only if \( 1 - z \geq x^*(\alpha, \pi) \).

**Proof:** See Appendix.

The proposition shows how the coalition behaves as a lender-of-last-resort by monitoring and by providing insurance.\(^9\) Monitoring corresponds to liquidating bad banks, those with the worst idiosyncratic shocks.

\(^9\) As is well known, in these types of models there are many (sunspot) equilibria, corresponding to other possible beliefs off the equilibrium path. This indeterminacy can be eliminated along the lines of Goldstein and Pauzner (1999), and Morris and Shin (2000) who show that adding even an infinitesimal amount of
idiosyncratic shock realizations. Member banks of type $r \in [0, x^*(\alpha, \pi)2M]$ are liquidated. These banks would have engaged in fraud. The insurance comes from the transfers implemented among the non-liquidated banks. Member banks of type $r \in [x^*(\alpha, \pi)2M, 2M]$ are not liquidated, but are assigned new debt obligations according to $D(\alpha, \pi, r) = \alpha + (1-f)(1+\beta-\alpha)(\pi+r)$. Their original debt, i.e., face value of the demand deposits, was one. Note that banks with $r < \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} - \pi$ have their debt reduced, i.e., $D(\alpha, \pi, r) < 1$, so these banks are subsidized to entice them not to engage in fraud. This is efficient because their projects are worth more if they are continued, as long as they do not engage in fraud. Member banks with $r > \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} - \pi$ have their debt increased, i.e., $D(\alpha, \pi, r) > 1$, so these banks are being taxed to pay the subsidy to the low $r$ banks. Banks with high idiosyncratic shock realizations cannot be taxed too much, or they will engage in fraud. The transfers of the debt obligations must satisfy the budget constraint that $\int_{x^*}^{2M} D(\alpha, \pi, r) dF(r) = 1$. This budget constraint limits how much insurance the coalition can provide and, therefore, determines the point at which member banks are liquidated.

Note that when the state of the economy is low and depositors run the banks, the number (measure) of member banks that the coalition has to liquidate depends on the state of the economy. When $\pi + M \geq \frac{1-\alpha}{(1-f)(1+\beta-\alpha)}$, there is no need for the coalition to liquidate any member banks. When $\pi + 2M < \frac{1-\alpha}{(1-f)(1+\beta-\alpha)}$, all the member banks have to be liquidated. (Note that since depositors can observe $\pi$ and $\alpha$, the coalition cannot pretend that it is not in a state where all members should be liquidated). When $\pi + M < \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} \leq \pi + 2M$, the coalition needs to liquidate some of the member banks.

There is a critical difference between how the coalition and the Big Bank deal with panics. The difference has to do with the difference between the ownership and property rights in these two systems. The banker of a Big Bank (implicitly) hires branch managers to manage branches for him, and he gets the entire surplus. We do not need to consider the branch managers’ incentives because the branch manager has no property rights over his branch. Consequently, when a Big Bank closes a branch, it gets $\alpha+(1+\beta-\alpha)Q-1$ after paying off the branch depositors and uses this private information eliminates the multiplicity of equilibria. For simplicity, we have specified the support
amount as additional reserves. These additional reserves change the incentives of the Big Bank. But, the coalition cannot increase reserves in this way because member banks have the property rights and hence control of their assets; they are free to quit the coalition. In order to entice them not to engage in fraud they must be rewarded. The payoff to a deviating bank is $\alpha + (1 + \beta - \alpha)Q - 1$, which is (weakly) dominated by the payoff it can get if it stays in the coalition.

At date 0, each bank must decide whether to join the coalition and the coalition must determine the optimal reserve level $\alpha$. The optimal reserve for the coalition is the solution of the following problem:

$$\text{Max}_\alpha \int_{\pi} \left[ \alpha + (1 + \beta - \alpha)xQ + (1 - x)(\pi + (1 + x)M) \right] dF(\pi) + \int_{\pi} \left[ \alpha + (1 + \beta - \alpha)(\pi + M) \right] dF(\pi) - 1$$

s.t. $\pi^r = \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)}$

$$x = \max \{0, \min \{1, \frac{1 - \alpha - \pi(1 - f)(1 + \beta - \alpha)}{M(1 - f)(1 + \beta - \alpha)} - 1\} \}.$$ 

$\alpha \in [0, \alpha_{\text{max}}]$

**Proposition 5:** The coalition’s objective function is strictly concave in $\alpha$. There is a unique optimal reserve level $\alpha \in [0, \alpha_{\text{max}}]$, that solves the coalition's optimization problem. At date 0, every bank strictly prefers to join the coalition.

**Proof:** See Appendix.

Intuitively, it is clear why each bank strictly prefers to join coalition. Joining the coalition is a verifiable act at date 0. The bank holds the specified level of reserves and capital and the coalition announces that the bank is a member. If the bank does not join the coalition at date 0, then it cannot join the coalition at date 1. If a bank is not in the coalition it is an independent Unit Bank and the proposition shows that this bank has lower expected profits than a coalition member.
The coalition system is an intermediate case between the Unit Bank system and the Big Bank system. The similarity between the coalition system and the independent Unit Bank system is that we may observe bank failures (i.e., liquidations) when the economy is in a bad state. The similarity between the coalition system and the Big Bank system is that the coalition can monitor and insure member banks when the economy is in a bad state, while the Big Bank “monitors” itself by closing branches. The one unique feature associated with the coalition is that when a panic occurs, it suspends convertibility and issues certificates. This feature is important because it is a commitment made to depositors that the non-liquidated member banks will not engage in fraud and it provides incentives for member banks to monitor and insure each other. The role of suspension of convertibility here is quite different from a coordination device used to eliminate Pareto-dominant equilibria in other models (e.g. Diamond and Dybvig (1983)).

C. Comparing the Different Bank Systems

We have studied three different banking systems: the independent Unit Banking system, the Big Bank system, and the bank coalition. In this section, we compare these systems in terms of welfare. Keep in mind that, on the one hand, holding reserves is inefficient because the risky project earns a higher return. But, on the other hand, holding fewer reserves means a higher chance of a bank panic, or of withdrawals in the case of the Big Bank system.

We first compare the optimal reserve levels and the likelihood of withdrawals under the different banking systems.

**Proposition 6:** The Unit Banking system holds more reserves than the coalition system, which, in turn, holds more reserves than the Big Bank system. Consequently, withdrawals are most likely in the Big Bank system, followed by the coalition system, and finally by the independent Unit Banking system.

**Proof:** See Appendix.

In the Unit Banking system, if depositors monitor banks by withdrawing, then the bank panic results in all banks being liquidated. Unit Banks are not diversified, nor do they have (private) deposit insurance like the coalition system. Banks in the Unit Banking system try to avoid the ex post losses from panics by holding more reserves. Banks in the Big Bank system can liquidate part of their assets to make a commitment that fraud is not going to happen. By closing some of the branches, the remaining projects can survive until date 2. Therefore, Big Banks invest more in the risky projects and hold less reserves. The coalition system lies between the Unit Banking
system and the Big Bank system. State contingent monitoring and co-insurance provide banks in the coalition with a way to survive panics if they are solvent. However, because the coalition system cannot completely replicate the Big Bank, banks in the coalition still hold more reserves than banks in the Big Bank system.

Proposition 6 leads to the important conclusion that there are more bank panics under the coalition system than under either of the other two organizational forms. Big Banks do not face panics. Though depositors’ withdrawals are the largest if they do monitor at date 1, Big Banks do not suspend convertibility, nor are they liquidated. In this sense, there is no panic. Unit Banks hold high reserves to reduce the likelihood of a date 1 panic. Banks in the coalition system hold fewer reserves than Unit Banks, but fewer banks are liquidated if there is a panic. Thus, the incidence of bank panics is a function of the organization of the banking system.

With respect to efficiency:

**Proposition 7:** The Big Bank system is more efficient than the coalition system, which is more efficient than the independent Unit Banking system.

**Proof:** See Appendix.

The Big Bank has two advantages. Unlike a Unit Bank, it is diversified, so the information asymmetry is eliminated. And second, it can close branches and use the proceeds as reserves to alter its incentives to engage in fraud. The coalition is diversified in the sense that the membership’s aggregate portfolio is diversified, but property rights in the coalition do not allow it to close member banks and use the proceeds as reserves for the remaining members.

Proposition 7 ranks the various banking systems in terms of welfare. Proposition 6 provides empirical predictions about banking system stability, that is failure or liquidation of banks. To illustrate the comparison of different banking systems, we present a numerical example. The parameters for the example are given in the first line of the box below.
Comparative Banking Systems: A Numerical Example

Assumed Parameters: $\pi_L=1.0$, $\pi_H=1.6$, $\beta=0.35$, $Q=0.75$, $f=0.4$, $M=0.06$.

<table>
<thead>
<tr>
<th></th>
<th>Unit Banks</th>
<th>Coalition</th>
<th>Big Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reserve Level Required for No Date 1 Withdrawals</strong></td>
<td>$\alpha^U_{\text{max}}=0.65$</td>
<td>$\alpha^B_{\text{max}}=0.60$</td>
<td>$\alpha^U_{\text{max}}=0.65$</td>
</tr>
<tr>
<td><strong>Equilibrium Reserve Level</strong></td>
<td>$\alpha^U=0.51$</td>
<td>$\alpha^C=0.32$</td>
<td>$\alpha^B=0.13$</td>
</tr>
<tr>
<td><strong>Bank Failures if Panic</strong></td>
<td>All Banks</td>
<td>Some Banks</td>
<td>No Banks</td>
</tr>
<tr>
<td><strong>Likelihood of Withdrawal at Date 1</strong></td>
<td>0.2</td>
<td>0.36</td>
<td>0.41</td>
</tr>
<tr>
<td><strong>Expected Value of Bank</strong></td>
<td>$\text{EV}^U=0.68$</td>
<td>$\text{EV}^C=0.78$</td>
<td>$\text{EV}^B=0.98$</td>
</tr>
</tbody>
</table>

The last row of the table confirms that the Big Bank system is the most efficient, followed by the coalition, and then the Unit Banks. Equilibrium reserves are highest in the Unit Bank system, followed by the coalition, and then by the Big Bank system. This corresponds to the comparison between the U.S. and Canada, shown in the previous table. Also consistent with that comparison is the observed level of failures. If there are withdrawals at date 1, there are no bank failures (liquidations) in the Big Bank system, but there are in the coalition system. The entire banking system fails when there is a panic in the Unit Bank system. As mentioned above, in Canada, the banking system shrank by the same order of magnitude as in the U.S. during the Great Depression, but there were no bank failures. In the U.S. during the Great Depression, the private bank coalition system did not exist anymore (following the 1914 establishment of the Federal Reserve System), and the entire banking system was insolvent, with roughly 30 percent liquidated.

**VII. Discussion**

We studied the industrial organization of banking. Banking systems with large, well-diversified, banks do not experience banking panics or failures. Banking panics occur in systems of small Unit Banks. Banking panics are not irrational. Panics result from depositors monitoring/liquidating banks in a setting where some banks are more likely to be engaging in
moral hazard, but the depositors do not know which banks are the more likely because of asymmetric information. The lender-of-last-resort function, including money creation, monitoring, and deposit insurance arose from private arrangements among banks. Bank coalitions formed to monitor members and provide insurance to depositors. Banking panics play a crucial role in making such private bank coalitions work. Because of the panic banks are forced to commit to pool resources and liquidate some members.

Note that the model could just as easily be interpreted as a model of the existence of banks. The Unit Banking system can be thought of as a system of small firms, while the Big Bank system can be thought of as a Large Firm. Again, these are two benchmarks. Firms issue debt because this allows them to be monitored. The coalition can now be thought of as a bank and the analysis demonstrates the role of intermediaries. In either case, the key is the delegation of monitoring to a coalition/intermediary, but this depends upon the “panic” inducing the coalition/intermediary to monitor its members/borrowers.

Why did government central banks replace private bank coalitions? In the above analysis, there is no obvious rationale for the government to step in and provide the lender-of-last-resort function unless the government has much more power than private agents, more resources than private agents, or there are costs to panics that have not been considered. Gorton and Huang (2001) consider the above model, but include a transactions role for bank liabilities. A panic disrupts the role of bank liabilities as a medium of exchange. They argue that in this context the government may be able to improve welfare with deposit insurance.
Appendix

Proof of Proposition 1: The bankers solve the following optimization problem at date 0:

\[
\text{Max } \alpha \int_{\pi_L}^{\pi_R} \left[ \alpha + (1+\beta-\alpha)(\pi - 1)\right]dF(\pi) + \int_{\pi_L}^{\pi_H} \left[ \alpha + (1+\beta-\alpha)(\pi + M - 1)\right]dF(\pi)
\]

s.t. \[\pi^* = \frac{1-\alpha}{f(1+\beta-\alpha)}\]
\[\alpha \in [0, \alpha_{\text{max}}].\]

Define \(EV(\alpha) = \int_{\pi_L}^{\pi_R} \alpha + (1+\beta-\alpha)QdF(\pi) + \int_{\pi_L}^{\pi_H} \alpha + (1+\beta-\alpha)(\pi + M)dF(\pi)\)

\[= \alpha + (1+\beta-\alpha)\left(\frac{\pi_L + \pi_H}{2} + M\right) - \int_{\pi_L}^{\pi_R} (1+\beta-\alpha)(\pi + M - Q) dF(\pi),\]

and \(C^U = 1 - \left(\frac{\pi_L + \pi_H}{2} + M\right) + A^2 \left(\frac{1}{1-f} - \frac{1}{1-f} - \pi_L\right)(Q - M) - \frac{1}{2}\pi_L^2.\) Then:

\[\frac{dEV(\alpha)}{d\alpha} = C^U - \frac{A^2}{2(1-f)^2(1+\beta-\alpha)^2}\]

and \(\frac{d^2EV(\alpha)}{d\alpha^2} = -\frac{A^2}{(1-f)^2(1+\beta-\alpha)^3} < 0.\)

Therefore \(EV(\alpha)\) is a strictly concave function of \(\alpha,\) and it has a unique maximum value in the interval \([0, \alpha_{\text{max}}^U].\) If \(\frac{dEV(0)}{d\alpha} > 0\) and \(\frac{dEV(\alpha_{\text{max}}^U)}{d\alpha} < 0,\) then we have an interior solution. //

Proof of Corollary 1:

\[\frac{d\alpha^U}{d\beta} = 1 - \frac{1}{(1-f)^2} \sqrt{\frac{A^2}{2C^U}} > 0\]
when \(\frac{A^2}{2(1-f)^2(1+\beta)^2} < C^U < \frac{A^2}{2(1-f)^2(1+\beta-\alpha_{\text{max}}^U)^2}.\) //
Proof of Proposition 2: A Big Bank solves the following optimization problem at date 0:

\[
\begin{align*}
\text{Max } & \int_{\pi_L}^{\pi_R} \alpha \left[ \alpha + (1 + \beta - \alpha)Q - 1 \right] dF(\tilde{\pi}) + \int_{\pi_L}^{\pi_H} \alpha \left[ \alpha + (1 + \beta - \alpha)(\pi + M) - 1 \right] dF(\tilde{\pi}) \\
\text{s.t. } & \pi^R + M = \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)} \\
& \alpha \in [0, \alpha_{B_{\text{max}}}].
\end{align*}
\]

Define \( EV(\alpha) = \int_{\pi_L}^{\pi_R} \alpha + (1 + \beta - \alpha)QdF(\tilde{\pi}) + \int_{\pi_L}^{\pi_H} \alpha + (1 + \beta - \alpha)(\pi + M)dF(\tilde{\pi}) \)

\[
= \alpha + (1 + \beta - \alpha)(\frac{\pi_L + \pi_H}{2} + M) - \int_{\pi_L}^{\pi_H} (1 + \beta - \alpha)(\pi + M - Q) dF(\tilde{\pi}) ,
\]

and \( C^B = 1 - (\frac{\pi_L + \pi_H}{2} + M) + A(\frac{1}{1 - f} - M)^2 - (\frac{1}{1 - f} - M - \pi_L)(Q - M) - \frac{1}{2} \frac{\beta^2}{\pi_L^2} \). Then:

\[
\frac{dEV(\alpha)}{d\alpha} = C^B - \frac{A\beta^2}{2(1 - f)^2(1 + \beta - \alpha)^2} \quad \text{and}
\]

\[
\frac{d^2EV(\alpha)}{d\alpha^2} = -\frac{A\beta^2}{(1 - f)^2(1 + \beta - \alpha)^3} < 0.
\]

Therefore \( EV(\alpha) \) is a strictly concave function of \( \alpha \), and it has a unique maximum value in interval \([0, \alpha_{\text{max}}^B] \). If \( \frac{dEV(0)}{d\alpha} > 0 \) and \( \frac{dEV(\alpha_{\text{max}}^B)}{d\alpha} < 0 \), then we have an interior solution. //

Proof of Lemma 3: Let \( G(x) = \alpha + (1 + \beta - \alpha) xQ + (1 + \beta - \alpha)(1 - f)(1 - x)(\pi + (1 + x)M) - 1 \).

\( G(x) \) is a continuous quadratic function of \( x \). Since \( G(0) < 0 \) and \( G(1) > 0 \), there exists a unique solution in the interval \([0, 1]\) for the equation \( G(x) = 0 \). The solution is the smaller root of the quadratic equation, which is:

\[
x = \frac{Q - (1 - f)\pi - \sqrt{(Q - (1 - f)\pi)^2 - \frac{4M(1 - f)(1 - \alpha)}{1 + \beta - \alpha} + 4M(1 - f)^2(\pi + M)}}{2M(1 - f)}. \quad \text{Note that}
\]
\[
\frac{4M(1-f)(1-\alpha)}{1+\beta-\alpha} = 4M(1-f)\frac{4M(1-f)\beta}{1+\beta-\alpha}
\]
is the only term through which \(\alpha\) and \(\beta\) affect \(x\), and it is decreasing in both \(\alpha\) and \(\beta\). \(x\) is increasing in \(\frac{4M(1-f)(1-\alpha)}{1+\beta-\alpha}\), and therefore decreasing in \(\alpha\) and \(\beta\).

To show \(x\) is decreasing in \(\pi\), let us define:

\[
Y = (Q - (1-f)\pi)^2 - \frac{4M(1-f)(1-\alpha)}{1+\beta-\alpha} + 4M(1-f)^2(\pi + M).\]

Then:

\[
\frac{\partial x}{\partial \pi} = -\frac{1}{2M} - \frac{1}{2} Y (Q - (1-f)^2 - (Q - (1-f)\pi + Y)^2).
\]

Since \(x < 1\), we have \(2M(1-f) - (Q - (1-f)\pi > -\frac{1}{2}\). Thus we have \(\frac{\partial x}{\partial \pi} < 0\). \(\square\)

**Proof of Proposition 3:** The Big Bank’s problem at date 0 is:

\[
\text{Max } \alpha \int_{\pi_L}^{\pi^r} [\alpha + (1+\beta-\alpha)xQ + (1+\beta-\alpha)(1-x)(\pi + (1+x)M) - 1]dF(\bar{\pi}) +
\]

\[
\int_{\pi_L}^{\pi_H} [\alpha + (1+\beta-\alpha)(\pi + M) - 1]dF(\bar{\pi})
\]

s.t. \(\pi^r + M = \frac{1-\alpha}{(1-f)(1+\beta-\alpha)}\)

\[
x = \frac{Q - (1-f)\pi - \sqrt{(Q - (1-f)\pi)^2 - \frac{4M(1-f)(1-\alpha)}{1+\beta-\alpha} + 4M(1-f)^2(\pi + M)}}{2M(1-f)}
\]

\(\alpha \in [0, \alpha_B^{\text{max}}]\)

Define \(\text{EV}(\alpha) = \int_{\pi_L}^{\pi^r} \alpha + (1+\beta-\alpha)xQ + (1+\beta-\alpha)(1-x)(\pi + (1+x)M)dF(\bar{\pi}) +
\]

\[
\int_{\pi_L}^{\pi_H} \alpha + (1+\beta-\alpha)(\pi + M)dF(\bar{\pi})
\]
\[ = \alpha + (1 + \beta - \alpha) \left( \frac{\pi_L + \pi_H}{2} + M \right) - \int_{\pi_L}^{\pi^*} (1 + \beta - \alpha) x(\pi + xM - Q) \, dF(\pi) \]

and rewrite \( x(\alpha) = C_0 - (C_1 + \frac{C_2}{1 + \beta - \alpha}) \frac{1}{2} \) where

\[ C_0 = \frac{Q - (1 - f)\pi}{2M(1 - f)} , \quad C_1 = \frac{(Q - (1 - f)\pi)^2 - 4M(1 - f) + 4M(1 - f)^2 (\pi + M)}{4M^2(1 - f)^2} \]

and

\[ C_2 = \frac{4M(1 - f)\beta}{M(1 - f)} . \]

We have \( \frac{\partial x}{\partial \alpha} = -\frac{C_2}{2} (C_1 + \frac{C_2}{1 + \beta - \alpha}) \frac{1}{2} (1 + \beta - \alpha)^{-2} < 0 \) and

\[ \frac{\partial^2 x}{\partial \alpha^2} = \frac{C_2^2}{4} (C_1 + \frac{C_2}{1 + \beta - \alpha}) \frac{3}{2} (1 + \beta - \alpha)^{-4} - C_2 (C_1 + \frac{C_2}{1 + \beta - \alpha}) \frac{1}{2} (1 + \beta - \alpha)^{-3} \]

\[ = \frac{C_2^2}{4} (C_1 + \frac{C_2}{1 + \beta - \alpha}) \frac{3}{2} (1 + \beta - \alpha)^{-4} + 2(1 + \beta - \alpha)^{-1} \frac{\partial x}{\partial \alpha} . \] Then:

\[ \frac{dEV(\alpha)}{d\alpha} = 1 - \left( \frac{\pi_L + \pi_H}{2} + M \right) + \int_{\pi_L}^{\pi^*} x(\pi + xM - Q) - \frac{\partial x}{\partial \alpha} (1 + \beta - \alpha)(\pi + 2xM - Q) \, dF(\pi) \]

\[ \frac{d^2 EV(\alpha)}{d\alpha^2} = \int_{\pi_L}^{\pi^*} (2 \frac{\partial x}{\partial \alpha} - (1 + \beta - \alpha) \frac{\partial^2 x}{\partial \alpha^2}) (\pi + 2xM - Q) \, dF(\pi) - \]

\[ \int_{\pi_L}^{\pi^*} 2M(1 + \beta - \alpha) \left( \frac{\partial x}{\partial \alpha} \right)^2 dF(\pi) + \frac{\beta}{(1 - f)(1 + \beta - \alpha)} (\pi^* - Q) \frac{\partial x}{\partial \alpha} . \]

Because \( \frac{\partial x}{\partial \alpha} < 0, \quad \frac{\partial x}{\partial \alpha} - (1 + \beta - \alpha) \frac{\partial^2 x}{\partial \alpha^2} < 0, \quad \text{and} \quad \pi + 2Mx - Q > 0, \) we have

\[ \frac{d^2 EV(\alpha)}{d\alpha^2} < 0. \] Therefore \( EV(\alpha) \) is a strictly concave function of \( \alpha \), and it has a unique maximum value in interval \([0, \alpha_{\text{max}}^\theta]\). If \( \frac{dEV(\alpha)}{d\alpha} > 0 \) and \( \frac{dEV(\alpha_{\text{max}}^\theta)}{d\alpha} < 0 \), then we have an interior solution. //
Proof of Lemma 4: Suppose each bank holds reserve level $\alpha$ and that the state of the macroeconomy is $\pi < \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)}$, i.e., there is a panic. Let us consider any measure of banks with any distribution function $F(\tilde{r})$ of idiosyncratic shocks in the interval $[\pi, \pi + 2M]$.

Define $r^* = \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)} - \pi$. We have $f(1 + \beta - \alpha)(\pi + r) > \alpha + (1 + \beta - \alpha)(\pi + r) - 1$ for $r < r^*$, i.e., these bank types will engage in fraud; $f(1 + \beta - \alpha)(\pi + r) = \alpha + (1 + \beta - \alpha)(\pi + r) - 1$ for $r = r^*$; and $f(1 + \beta - \alpha)(\pi + r) < \alpha + (1 + \beta - \alpha)(\pi + r) - 1$ for $r > r^*$, i.e., these bank types will not engage in fraud. If there is no bank run, the banks with $r < r^*$ engage in fraud and the banks with $r \geq r^*$ do not engage in fraud. The total payoff to all the banks is:

$$\int_0^{r^*} f(1 + \beta - \alpha)(\pi + r) dF(\tilde{r}) + \int_{r^*}^{2M} \left[ \alpha + (1 + \beta - \alpha)(\pi + r) - 1 \right] dF(\tilde{r}).$$

If these banks pool their liabilities, under any system of transfers among members, then they can be either solvent or insolvent at date 2. If they are solvent, the total of their individual payoffs cannot exceed $\int_0^{2M} \left[ \alpha + (1 + \beta - \alpha)(\pi + r) - 1 \right] dF(\tilde{r})$, which can be reached when none of these banks engage in fraud. If they are insolvent, the total of their payoffs cannot exceed $\int_0^{2M} f(1 + \beta - \alpha)(\pi + r) dF(\tilde{r})$, which can be reached when all these banks engage in fraud.

Therefore, the maximum total payoff from pooling under any system of transfers is:

$$\max \left\{ \int_0^{2M} \left[ \alpha + (1 + \beta - \alpha)(\pi + r) - 1 \right] dF(\tilde{r}), \int_0^{2M} f(1 + \beta - \alpha)(\pi + r) dF(\tilde{r}) \right\}.$$ 

Since $\int_0^{r^*} f(1 + \beta - \alpha)(\pi + r) dF(\tilde{r}) + \int_{r^*}^{2M} \left[ \alpha + (1 + \beta - \alpha)(\pi + r) - 1 \right] dF(\tilde{r}) 

\geq \max \left\{ \int_0^{2M} \left[ \alpha + (1 + \beta - \alpha)(\pi + r) - 1 \right] dF(\tilde{r}), \int_0^{2M} f(1 + \beta - \alpha)(\pi + r) dF(\tilde{r}) \right\}$, it is better for these banks choose not to pool their liabilities in a coalition. That is, as a group it is a dominant strategy to deviate from any proposed coalition.

Hence we have shown it would be better for the non-liquidated banks to stay independent Unit Banks if there is no bank panic. For those banks that are supposed to be liquidated, the bankers can get more from engaging in fraud than being liquidated. Since the coalition maximizes the total payoffs to all member bankers, it is not going to operate without a bank panic. //
Proof of Proposition 4: At date 1, depositors run the banks when \( \pi < \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} \). The coalition has to operate, choose \( P(\alpha, \pi)=1 \), to save as many member banks as possible from being liquidated. The coalition starts the monitoring and coinsurance system. First, it can liquidate some member banks to improve the distribution of idiosyncratic shocks of those remaining banks. Second, it can implement the system of transfers to pool liabilities. We restrict the coalition’s actions to two functions: \( L(\alpha, \pi, r) \) and \( D(\alpha, \pi, r) \). \( L(\alpha, \pi, r) \) is a mapping from \([0, 2M]\) to \(\{1, 0\}\), indicating whether a member bank (in state \( \{\alpha, \pi\} \)) with idiosyncratic shock \( r \) is to be liquidated \((L=1)\) or not liquidated \((L=0)\). If the member is not liquidated, then the second rule, \( D(\alpha, \pi, r) \), which is a mapping from \([0, 2M]\) to \(\mathbb{R}^+\), indicates the liability reallocated to a member bank with idiosyncratic shock \( r \).

We first determine the best possible outcomes for the coalition. Then we prove the equilibrium characterized in Proposition 4 generates the best possible outcome.

If the coalition has to liquidate some of the member banks, it is always better to liquidate those banks with low realizations of idiosyncratic shocks. This is because all the risky projects have the same liquidation value. For the liquidated member banks, the coalition gives the bankers \( \alpha+(1+\beta-\alpha)Q-1 \). On the one hand, the coalition has to pay them at least \( \alpha+(1+\beta-\alpha)Q-1 \), the worst possible payoff they can get if they quit; on the other hand, the coalition needs not to pay them more than \( \alpha+(1+\beta-\alpha)Q-1 \), since the incentives of these bankers no longer need to be considered.

Now let us considered the non-liquidated member banks. Suppose a fraction, \( x \), of banks with low idiosyncratic shocks are liquidated. These will be the banks with the lowest shock realizations, so the remaining banks will have idiosyncratic shocks distributed in the interval: \([x2M, 2M]\), which has a mean of \((1+x)M\). According to Assumption 13, the coalition needs the original bankers to operate these banks. In order to prevent a banker with idiosyncratic shock \( r \) from engaging in fraud, the coalition has to promise him a payoff of at least \( f(1+\beta-\alpha)(\pi+r) \). Therefore, in order to convince depositors that their deposits are safe if they accept clearinghouse loan certificates, the coalition has to satisfy the following condition:

\[
\int_{x2M}^{2M} (\alpha+(1+\beta-\alpha)(\pi+r)-1)dF(\tilde{r}) \geq \int_{x2M}^{2M} f(1+\beta-\alpha)(\pi+r)dF(\tilde{r}).
\]

Solving for \( x \), we can rewrite the condition as:

\[
x \geq \frac{1-\alpha-\pi(1-f)(1+\beta-\alpha)}{M(1-f)(1+\beta-\alpha)} - 1.
\]

The fraction \( x \) is between 0 and 1. Therefore, imposing this condition, in order to convince depositors that the remaining banks
have no incentives to engage in fraud, the coalition must liquidate a fraction, \( x \), of the member banks such that
\[
x \geq x^* (\alpha, \pi) \equiv \max \{0, \min \left\{ \frac{1 - \alpha - \pi(1 - f)(1 + \beta - \alpha)}{M(1 - f)(1 + \beta - \alpha)} - 1 \right\} \}.
\]

The coalition’s task is now reduced to finding the minimum \( x \) that can be accepted by the depositors. According to the proposition, in equilibrium the coalition only needs to liquidate exactly \( x^* (\alpha, \pi) \) of banks. We need to check whether the coalition’s sharing rules can prevent the non-liquidated member banks from engaging in fraud and depositors’ beliefs satisfying equilibrium requirements. Moreover, we need to check whether depositors’ off equilibrium path beliefs ensure that the coalition has no profitable deviation.

First, non-liquidated member banks have no incentive to engage in fraud. A non-liquidated member bank with idiosyncratic shock of \( r \) owes a liability of \( D(\alpha, \pi, r) = \alpha + (1 - f)(1 + \beta - \alpha)(\pi + r) \) to the coalition. If it does not engage in fraud and honors its liabilities, its payoff is
\[
\alpha + (1 + \beta - \alpha)(\pi + r) - \{ \alpha + (1 - f)(1 + \beta - \alpha)(\pi + r) \} = f(1 + \beta - \alpha)(\pi + r),
\]
which is what it can get from engaging in fraud. Thus, it has no incentive to engage in fraud.

Second, depositors observe all the banks remain with the coalition and a fraction \( x^* (\alpha, \pi) \) of member banks are liquidated. To be consistent with the actions taken by the coalition and its members, their beliefs must be that banks with idiosyncratic shocks in the interval: \([0, x^* (\alpha, \pi)2M]\) are liquidated and banks with idiosyncratic shocks in the complementary interval, \([x^* (\alpha, \pi)2M, 2M]\), are not liquidated and have no incentive to engage in fraud. According to their updated beliefs, their deposits are safe and they accept the certificates. We need to specify depositors’ off equilibrium path beliefs. Suppose depositors observe a fraction \( y \) of banks quit the coalition, a fraction \( z \) of the banks remain non-liquidated in the coalition and a fraction \( 1 - y - z \) of banks are liquidated by the coalition. They believe that the idiosyncratic shocks of those banks out of the coalition are distributed in the interval: \([0, y2M]\), and the idiosyncratic shocks of those non-liquidated banks in the coalition are distributed in the interval: \([(1 - z)2M, 2M]\).

Finally, we check that the coalition has no profitable deviation. There are two possible deviations for the coalition. The coalition can liquidate a fraction of member banks less than \( x^* (\alpha, \pi) \). Or it can carry out a different set of sharing rules. According to depositors’ off equilibrium path beliefs, if the coalition liquidates a fraction of banks less than \( x^* (\alpha, \pi) \), the certificates will not be accepted by depositors and all the member banks will be liquidated. Therefore, this is not a profitable deviation. On the other hand, no matter what kind of sharing rules are carried out inside the coalition, the coalition can either be solvent or insolvent at date 2. If the coalition is solvent at date 2, its maximum payoff is: \((1-\)
\(x^*(\alpha, \pi)(\alpha+(1+\beta-\alpha)(\pi+(1+M)x^*(\alpha, \pi))^{-1})\), which can be reached when none of the remaining member banks engage in fraud. If the coalition is insolvent due to some members engaging in fraud, its maximum payoff is: \((1-x^*(\alpha, \pi))f(1+\beta-\alpha)(\pi+(1+M)x^*(\alpha, \pi))^{-1}\), reached when all the non-liquidated banks engage in fraud. Since \((\alpha+(1+\beta-\alpha)(\pi+(1+M)x^*(\alpha, \pi))^{-1})= f(1+\beta-\alpha)(\pi+(1+M)x^*(\alpha, \pi))^{-1}\) if \(x^*(\alpha, \pi)<1\), the coalition has no profitable deviation from the specified sharing rules. \\

**Proof of Proposition 5:** The coalition’s objective function is:

\[
\text{Max}_{\alpha} \\
\int_{\pi}^{\pi'} [\alpha + (1 + \beta - \alpha) x Q + (1 - x)(\pi + (1 + M)) - 1]dF(\pi) + \int_{\pi'}^{\pi''} [\alpha + (1 + \beta - \alpha)(\pi + M) - 1]dF(\pi)
\]

s.t. \(\pi^r = \frac{1-\alpha}{(1-f)(1+\beta-\alpha)}\)

\(x = \max\{0, \min\{1, \frac{1-\alpha-\pi(1-f)(1+\beta-\alpha)}{M(1-f)(1+\beta-\alpha)}-1\}\}\).

\(\alpha \in [0, \alpha^*_{\text{max}}]\)

First, if \(\alpha \geq \alpha^*_{\text{max}}\), then \(x=0\) for any realization of \(\pi\). So, we only need to consider the case where \(\alpha \in [0, \alpha^*_{\text{max}}]\). Define \(\alpha^* = \frac{1-(1+\beta)(1-f)(\pi L + 2M)}{1-(1-f)(\pi L + 2M)}\). If \(\alpha > \alpha^*\), then \(x\) is always less than 1, which means that there is no need to liquidate all the member banks. Then the objective function can be written as:

\[
\text{EV}^B(\alpha) = \text{EV}^1(\alpha) = \int_{\pi L}^{\pi^r} \alpha + (1 + \beta - \alpha)x Q + (1 + \beta - \alpha)(1 - x)(\pi + (1 + M))dF(\pi) + \\
\int_{\pi^r}^{\pi^H} \alpha + (1 + \beta - \alpha)(\pi + M)dF(\pi)
\]

= \(\alpha + (1 + \beta - \alpha)(\frac{\pi L + \pi H}{2} + M) - \int_{\pi L}^{\pi^r} \alpha + (1 + \beta - \alpha)x(\pi + M - Q)dF(\pi)\)

If \(\alpha \leq \alpha^*\), then \(x\) is equal to 1 when \(\pi \leq \frac{1-\alpha}{(1-f)(1+\beta-\alpha)} - 2M\), which means the coalition needs to liquidate all the member banks when \(\pi\) is low enough. Then the objective function can be written as:
EV^2(\alpha) = EV^2(\alpha) = \int_{\pi_L}^{\pi_H} \alpha + (1 + \beta - \alpha)QdF(\bar{\pi}) +

\int_{\pi}^{\pi^*} \alpha + (1 + \beta - \alpha)xQ + (1 + \beta - \alpha)(1 - x)(\pi + (1 + x)M)dF(\bar{\pi}) +

\int_{\pi}^{\pi^*} \alpha + (1 + \beta - \alpha)(\pi + M)dF(\bar{\pi})

= \alpha + (1 + \beta - \alpha)(\frac{\pi_L + \pi_H}{2} + M) - \int_{\pi}^{\pi^*} (1 + \beta - \alpha)(\pi + M - Q)dF(\bar{\pi}) - \int_{\pi}^{\pi^*} (1 + \beta - \alpha)x(\pi + xM - Q)dF(\bar{\pi}).

Next, we show both EV^1(\alpha) and EV^2(\alpha) are concave and \( \frac{dEV^1(\alpha)}{d\alpha} = \frac{dEV^2(\alpha)}{d\alpha} \) when \( \alpha = \alpha^* \).

\[
\frac{dEV^1(\alpha)}{d\alpha} = 1 - (\frac{\pi_L + \pi_H}{2} + M) + \int_{\pi}^{\pi^*} \frac{x(\pi + xM - Q) - \frac{\partial^2 x}{\partial \alpha^2}}{(1 - f)(1 + \beta - \alpha)}(\pi + 2xM - Q)dF(\bar{\pi}) -
\]

\[
\int_{\pi}^{\pi^*} 2M(1 + \beta - \alpha)\frac{\partial x}{\partial \alpha}dF(\bar{\pi}) + \frac{\beta}{(1 - f)(1 + \beta - \alpha)}(\pi - Q)\frac{\partial x}{\partial \alpha}.
\]

Since \( \frac{\partial x}{\partial \alpha} = -\frac{\beta}{M(1 - f)}(1 + \beta - \alpha)^{-2} < 0, \frac{\partial^2 x}{\partial \alpha^2} = -\frac{2\beta}{M(1 - f)}(1 + \beta - \alpha)^{-3} < 0, \) we have

\[
2\frac{\partial x}{\partial \alpha} - (1 + \beta - \alpha)\frac{\partial^2 x}{\partial x^2} = 0, \text{ and thus } \frac{d^2EV^1(\alpha)}{d\alpha^2} < 0.
\]

\[
\frac{dEV^2(\alpha)}{d\alpha} = 1 - (\frac{\pi_L + \pi_H}{2} + M) + \int_{\pi}^{\pi^*} (\pi + M - Q)dF(\bar{\pi}) +
\]

\[
\int_{\pi}^{\pi^*} \frac{x(\pi + xM - Q) - \frac{\partial x}{\partial \alpha}}{(1 + \beta - \alpha)(\pi + 2xM - Q)dF(\bar{\pi}) -
\]

\[
\int_{\pi}^{\pi^*} 2M(1 + \beta - \alpha)\frac{\partial x}{\partial \alpha}dF(\bar{\pi}) + \frac{\beta}{(1 - f)(1 + \beta - \alpha)}(\pi - M - Q)\frac{\partial x}{\partial \alpha} < 0.
\]
Moreover when \( \alpha = \alpha^*, \pi^* = \pi_L \), we have \( \text{EV}^1(\alpha^*) = \text{EV}^2(\alpha^*) \), and \( \frac{d\text{EV}^1(\alpha^*)}{d\alpha} = \frac{d\text{EV}^2(\alpha^*)}{d\alpha} \).

Therefore, \( \text{EV}^B(\alpha) \) is a strictly concave function of \( \alpha \) and there is a unique \( \alpha \in [0, \alpha_{\text{max}}^B] \) that solves the coalition’s optimization problem.

To prove that all the banks are willing to join the coalition, we need to show that the expected payoff from joining the coalition is larger than that from operating as an independent bank. This is part of the proof of Proposition 6, below. //

**Proof of Proposition 6:** Since we have shown that the objective functions are concave, it suffices to show that

\[
\frac{d\text{EV}^U(\alpha)}{d\alpha} > \frac{d\text{EV}^C(\alpha)}{d\alpha} > \frac{d\text{EV}^B(\alpha)}{d\alpha}.
\]

Let \( \pi^* = \frac{1 - \alpha}{(1 - f)(1 + \beta - \alpha)} \),

\[
\frac{d\text{EV}^U(\alpha)}{\alpha} = 1 - \left( \frac{\pi_L + \pi_H}{2} + M \right) + \int_{\pi_L}^{\pi^*} (\pi + M - Q) \, dF(\pi) \cdot \frac{d\pi^*}{d\alpha} (\pi^* + M - Q)
\]

\[
\frac{d\text{EV}^C(\alpha)}{\alpha} = 1 - \left( \frac{\pi_L + \pi_H}{2} + M \right) + \int_{\pi_L}^{\pi^*} x^C (\pi + x^C M - Q) \, dF(\pi)
\]

\[- \int_{\pi_L}^{\pi^*} \frac{\partial x^C}{\partial \alpha} (1 + \beta - \alpha)(\pi + 2x^C M - Q) \, dF(\pi)
\]

\[
\frac{d\text{EV}^B(\alpha)}{\alpha} = 1 - \left( \frac{\pi_L + \pi_H}{2} + M \right) + \int_{\pi_L}^{\pi^*} x^B (\pi + x^B M - Q) \, dF(\pi)
\]

\[- \int_{\pi_L}^{\pi^*} \frac{\partial x^B}{\partial \alpha} (1 + \beta - \alpha)(\pi + 2x^B M - Q) \, dF(\pi)
\]

Since we have \( x^B < x^C < 1 \) and \( \frac{\partial x^B}{\partial \alpha} < \frac{\partial x^C}{\partial \alpha} < 0 \), we have

\[
\frac{d\text{EV}^U(\alpha)}{d\alpha} > \frac{d\text{EV}^C(\alpha)}{d\alpha} > \frac{d\text{EV}^B(\alpha)}{d\alpha}.
\]

Therefore \( \alpha^B < \alpha^C < \alpha^U \).

**Proof of Proposition 7:** Given \( \alpha \) and \( \pi \), we use \( x^B(\alpha, \pi), x^C(\alpha, \pi), \) and \( x^U(\alpha, \pi) \) to denote the fraction of banks (or projects) that need to be liquidated under the Big Bank system, the coalition, and the Unit Bank system, respectively. And we use \( \text{EV}^B(\alpha, \pi), \text{EV}^C(\alpha, \pi), \) and \( \text{EV}^U(\alpha, \pi) \) to denote the average payoff, respectively, to a bank (or project) over the distribution of idiosyncratic shock \( r \) under these three systems.
When $\pi \geq \frac{1 - \alpha}{1 - f(1 + \beta - \alpha)}$, $x^B(\alpha, \pi) = x^C(\alpha, \pi) = x^U(\alpha, \pi) = 0$, so there is no need to liquidate any banks (projects) under any of the three systems. Therefore, $EV_B(\alpha, \pi) = EV_C(\alpha, \pi) = EV_U(\alpha, \pi) = \alpha + (1 + \beta - \alpha)(\pi + M) - 1$.

When $\frac{1 - \alpha}{1 - f(1 + \beta - \alpha)} - M \leq \pi < \frac{1 - \alpha}{1 - f(1 + \beta - \alpha)}$, $x^B(\alpha, \pi) = x^C(\alpha, \pi) = 0$, $x^U(\alpha, \pi) = 1$, the independent Unit Banks are liquidated, while there is no need to liquidate any banks (projects) under the Big Bank system or the coalition system. $EV_B(\alpha, \pi) = EV_C(\alpha, \pi) = \alpha + (1 + \beta - \alpha)(\pi + M) - 1$, $EV_C(\alpha, \pi) = \alpha + (1 + \beta - \alpha)Q - 1$. Therefore we have $EV_B(\alpha, \pi) = EV_C(\alpha, \pi) > EV_C(\alpha, \pi)$.

When $\pi < \frac{1 - \alpha}{1 - f(1 + \beta - \alpha)} - M$, $x^B(\alpha, \pi) < x^C(\alpha, \pi) \leq x^U(\alpha, \pi) = 1$, there are liquidations under all the three systems. All the independent unit banks have to be liquidated. Only a fraction of the banks (projects) need to be liquidated under the Big Bank system and the coalition system. Note that:

$x^C(\alpha, \pi)$ solves: $(1 + \beta - \alpha)(\pi + (1 + x)M) - 1 = f(1 + \beta - \alpha)(\pi + (1 + x)M)$.

$x^B(\alpha, \pi)$ solves:

$\alpha + (1 + \beta - \alpha)xQ + (1 + \beta - \alpha)(1 - x)((\pi + (1 + x)M) - 1 = f(1 + \beta - \alpha)(1 - x)(\pi + (1 + x)M)$.

Since $\alpha + (1 + \beta - \alpha)x^C(\alpha, \pi)Q + (1 + \beta - \alpha)(1 - x^C(\alpha, \pi))(\pi + (1 + x^C(\alpha, \pi)M) - 1 = \alpha + (1 + \beta - \alpha)x^C(\alpha, \pi)Q + f(1 + \beta - \alpha)(1 - x^C(\alpha, \pi))M$, we have $x^B(\alpha, \pi) < x^C(\alpha, \pi)$.

$EV_B(\alpha, \pi) = \alpha + x^B(\alpha, \pi)(1 + \beta - \alpha)Q + (1 - x^B(\alpha, \pi))(1 + \beta - \alpha)(\pi + (1 + \beta - \alpha)M) - 1$,

$EV_C(\alpha, \pi) = \alpha + x^C(\alpha, \pi)(1 + \beta - \alpha)Q + (1 - x^C(\alpha, \pi))(1 + \beta - \alpha)(\pi + (1 + \beta - \alpha)M) - 1$,

$EV_U(\alpha, \pi) = \alpha + (1 + \beta - \alpha)Q - 1$.

Therefore, we have $EV_B(\alpha, \pi) > EV_C(\alpha, \pi) \geq EV_U(\alpha, \pi)$ when $\pi < \frac{1 - \alpha}{1 - f(1 + \beta - \alpha)} - M$.

Since we have $EV_B(\alpha, \pi) \geq EV_C(\alpha, \pi) \geq EV_U(\alpha, \pi)$ for all $\alpha$ and $\pi$, and $EV_B(\alpha, \pi) > EV_C(\alpha, \pi) > EV_U(\alpha, \pi)$ for some $\alpha$ and $\pi$, the Big Bank system dominates the coalition, which dominates the Unit Bank system. //
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