Purchasing, Pricing, and Quick Response in the Presence of Strategic Consumers

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We consider a retailer that sells a product with uncertain demand over a finite selling season. The retailer sets an initial stocking quantity and, at some predetermined point in the season, optimally marks down remaining inventory. We modify this classic setting by introducing three types of consumers: myopic consumers, who always purchase at the initial full price; bargain-hunting consumers, who purchase only if the discounted price is sufficiently low; and strategic consumers, who strategically choose when to make their purchase. A strategic consumer chooses between a purchase at the initial full price with the possibility, if inventory remains, of a later purchase at a markdown price. In equilibrium, strategic consumers and the retailer make optimal decisions given their rational expectations regarding the availability of inventory, expected prices and the behavior of other consumers. We find that the retailer stocks less, takes smaller price discounts and earns lower profit if strategic consumers are present than if there are no strategic consumers. We find that a retailer should generally avoid committing to a price path over the season (assuming such commitment is feasible) - committing to a markdown price (or to not markdown at all) is often too costly (inventory may remain unsold) even in the presence of strategic consumers; the better approach is to be cautious with the initial quantity, and then markdown optimally. Our most notable finding is with respect to the value of quick response (the ability to receive a mid-season replenishment, albeit at a higher unit cost than the initial order). We find that under relatively mild conditions, the value of quick response to a retailer is much greater in the presence of strategic consumers than without them: on average 67% more valuable and as much as 558% more valuable, in our sample. In other words, although it is well established in the literature that quick response provides value by allowing better matching of supply with demand, it provides more value, often substantially more value, by allowing a retailer to control the negative consequences of strategic consumer behavior.

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1 Introduction

Retailers are increasingly cognizant of the fact that modern consumers are educated, sophisticated, and willing to go to extraordinary lengths to purchase goods at the lowest possible price (Silverstein and Butman 2006). One common and powerful tactic consumers use to achieve this goal is to wait to purchase items only when they are on sale or clearance, a strategy aided by the fact that many retailers have predictable seasonal markdown patterns and offer deep discounts (e.g., Warner and Barsky 1995 report an average markdown of 39% on men's sweaters over a four month period). Customers who behave in this manner are referred to in the academic literature as strategic or rational consumers: they are non-myopic utility maximizers who recognize that a desired product is likely to be reduced in price at some point in time and they take these future markdowns into account, along with the expected availability of the product, when timing their purchasing decisions. All too often, as a result, strategic consumers choose to wait for markdowns, thereby denying retailers full price sales (Rozhon 2004).

O'Donnell (2006) suggests retailers can employ the following two tactics to induce strategic consumers to purchase products at full price: limit quantities to create a sense of scarcity and promote affordable full prices. Perhaps no firm is more successful at implementing these tactics than Zara, the Spanish fashion retailer. Since its inception, Zara has recognized the importance of minimizing the number and severity of markdowns in its retail outlets. According to Luis Blanc, a director for Zara's parent company Inditex, Zara essentially tries to eliminate the value of a strategic consumer's option to buy at a markdown price (Ghemawat and Nueno 2003): "...most important, we want our customers to understand that if they like something, they must buy it now, because it won't be in the shops the following week. It is all about creating a climate of scarcity and opportunity." To execute this strategy, Zara monitors and replenishes inventory frequently in stores (as often as twice a week) and produces 85% of its in-house inventory during the fashion season in which it is sold, as compared to 0-20% at most of its competitors (Ghemawat and Nueno 2003). This quick response capability comes with a price: Zara produces much of its merchandise in Europe, which has relatively expensive labor compared to outsourced production in Asia, and Zara frequently expedites shipments via expensive transportation methods such as air freight (Ferdows et al. 2004).

In this paper we study the interaction between a retailer's stocking decision and its markdown strategy in the presence of strategic consumers. Strategic consumers choose between either buying an item early in the selling season at the full price or waiting until later in the season when the item may be marked down in price. Waiting for the potential deal has two drawbacks from the consumer's perspective: the strategic consumer values purchasing the item less at the end of the season than at the start of the season (e.g., a barbecue is more valuable at the start of summer than at the end of summer) and the item may be unavailable at the end of the season, i.e., there is an availability risk associated with waiting. Furthermore, availability and pricing are interconnected; if availability is high, the retailer is likely to offer a deep discount, whereas if inventory is limited (either because the retailer was conservative with her initial buy or because demand turns out to be greater than expected), the markdown will be modest, assuming the item is reduced in price at all. However, the potential benefit of waiting is clear: the markdown may indeed be substantial, thereby providing the consumer with a great value. For example, a strategic consumer may be willing to purchase a barbecue for $350 at the start of summer but prefer even more the chance to purchase it at the end of summer for 50% off the initial price. Thus, we seek to identify the set of rational expectation equilibria in our model: the retailer chooses an optimal order quantity and markdown price given her expectation of consumer behavior and consumers choose an optimal purchasing strategy given their expectation of the behavior of the retailer and the other consumers.

Based on our stylized model, we address several questions. Under what conditions is it optimal to restrict quantities when facing strategic consumers? How do strategic consumers influence a retailer's pricing strategy, both its initial price as well as the depth of its discounting? Should a retailer commit to a price path throughout the selling season (i.e., commit to a specific markdown price or to not mark down at all) or is the retailer better off with a dynamic pricing strategy that sets an optimal markdown given the available inventory and initial sales? What is the potential loss in profit if a retailer ignores strategic behavior? Finally, and most importantly, what is the value of quick response capabilities in the presence of strategic consumers? It is well known that quick response provides substantial value to a retailer when consumers are assumed to be entirely myopic (i.e., non-strategic) because quick response allows a retailer to exploit updated information to better match supply with uncertain demand: with a quick response capability the retailer makes smaller inventory investments to mitigate the consequences of left over inventory while using the ability to replenish to lessen the opportunity cost of lost sales. Is the incremental value of quick response greater or smaller in the presence of strategic consumers? The answer to this question is critical for understanding whether or not a firm should invest in quick response capabilities, such as Zara's investments in localized production and expedited shipments.

The remainder is the paper is organized as follows. Section 2 reviews the literature and §3 describes the model. Section 4 analyzes the retailer's profit function, while §5 addresses the consumer best response function and §6 examines the equilibrium of the game. Section 7 considers the value of quick response inventory practices, §8 addresses the retailer's initial price decision in addition to its markdown decision, §9 reports results from a numerical analysis, and §10 concludes with a summary of the answers to our research questions.

2 Related Literature

A wide variety of models characterized by supply and demand mismatches have recently emerged that explicitly incorporate consumer preferences or behavior. Examples include competing over price and service level for a random number of customers (Deneckere and Peck 1995), service level stimulating demand (Dana and Petruzzi 2001), capacity management via reservations in a restaurant facing uncertain demand (Alexandrov and Lariviere 2006), and strategic joining of queues by arriving customers (Veeraraghavan and Debo 2005). The most relevant of these models to our analysis are those concerning multiperiod pricing.

Multiperiod pricing models are generally characterized by firms selling to consumers with unknown or heterogeneous valuations. Inventories are typically fixed, but the firm has the ability to change the price over time, exploiting this ability to price discriminate between consumers or to discover information about their valuations. For example, Lazear (1986) explains a variety of observed retail pricing phenomena via a fixed-inventory, two-period pricing framework with myopic consumers who purchase if their valuation of the product exceeds the price.

The addition of strategic consumers to the dynamic pricing problem is addressed by Besanko and Winston (1990), who model an uncapacitated, monopolistic retailer selling to a fixed number of heterogeneous, rational consumers over an arbitrary number of periods. The presence of strategic consumers leads to lower prices in each period than would be optimal with myopic consumers, because the retailer competes intertemporally with itself (i.e., consumers have the option to wait until a later period to purchase). There is no uncertainty in the model, thus there is no risk of stock-outs or left over inventory. More recently, several paper analyze various aspects of the markdown problem with strategic consumers and fixed inventories, including multi-unit customer demand (Elmaghraby et al. 2006a), pre-announced markdown policies with reservations (Elmaghraby et al. 2006b), continuously declining consumer valuations (Aviv and Pazgal 2005), uncertain, evolving consumer valuations (Gallego and Sahin 2006), and heterogeneous consumer populations (Su 2005).

Several recent papers study how a retailer's initial stocking level is affected by strategic consumers. In each case the price path over the selling season is fixed (either exogenously set or chosen by the retailer at the start of the selling season). Yin and Tang (2006) compare the efficacy of two different in-store display formats to manipulate consumer expectations regarding the availability of inventory. (In our model the retailer can influence consumer expectations only via its order quantity decision.) Liu and van Ryzin (2005) find that if consumers are risk-averse, even if demand is deterministic, it is optimal for the retailer to create a rationing risk by understocking initially to induce consumers to purchase early. (In our model consumers are risk neutral.) Su and Zhang (2005) conclude that several types of contracts can coordinate the supply chain when consumers are strategic, including, surprisingly, wholesale price contracts.

Our model is distinct along two key dimensions. First, in our model the retailer chooses both an initial stocking level and its markdown price dynamically (and in an extension, the retailer sets the initial price as well). In other words, an optimal markdown is chosen given initial season sales and remaining inventory: a greater discount is offered if either initial sales are weak or if there is a substantial amount of inventory remaining. (Cachon and Kok 2002 study a similar model of optimal dynamic markdowns, but they do not consider the presence of strategic consumers.) Liu and van Ryzin (2005) assume the retailer commits to a price path for the season. They argue that a retailer may be able to credibly commit to a price path but they do not explicitly determine whether such a commitment would be desirable in their model. (Besanko and Winston 1990 find, in their model, that a retailer does benefit from a commitment to a price path.) Aviv and Pazgal (2005) do numerically evaluate whether a retailer is better off committing to a price path or choosing an optimal markdown price (given the initial fixed quantity of inventory) and find that commitment is generally better for the retailer. Dasu and Tong (2005) find that posted pricing schemes perform nearly optimally with fixed quantities, and are usually preferred to contingent pricing schemes. Our results are different: when a retailer is able to choose the initial stocking quantity and its markdown price, the retailer is generally better off dynamically pricing rather than committing to a markdown price. As a result, in our setting, even if a retailer could commit to a price path (e.g., due to repeated game dynamics), the retailer is generally not better off doing so.

The second key distinction of our analysis is that we investigate the value of quick response capabilities in the presence of strategic consumers. There is a broad literature documenting the large benefit of quick response in a supply chain (e.g., Barnes-Schuster et al. 2002; Eppen and Iyer 1997; Fisher and Raman 1996; Fisher et al. 2001; Iyer and Bergen 1997; Jones et al. 2001; Petruzzi and Dada 2001). However, to the best of our knowledge, the influence of strategic consumer behavior on the value of quick response has not been addressed.

3 Model Description

We model a single firm (the retailer) selling a single product over two periods. In the first period, the retailer sells the product at a fixed, exogenous full price p (we later relax this assumption). In the second period, the product is sold for the markdown price s. We refer to the first period as the "full price" period and the second period as the "sale" or "salvage" period.

The retailer has two decisions: sale price and initial stocking quantity. The sale price is chosen at the start of the second period to maximize revenue, $R(s, I)$, where I is the inventory available at the beginning of the second period. Prior to the first period, the stocking level q is chosen to maximize total expected profit, $\pi(q)$. We assume initially that production leadtimes are long enough that there is only one purchasing opportunity (this assumption is relaxed in §7).

The unit procurement cost to the retailer is c. Any inventory remaining at the end of the second period has zero value. The total number of customers that may purchase in the first period is a random variable $D \geq 0$ with distribution $F(\cdot)$, complementary cdf $\overline{F}(\cdot) = 1 - F(\cdot)$, and density $f(\cdot)$. We assume that D satisfies the following property.

Definition 1 *A continuous, non-negative random variable X with density f satisfies the* ***monotone scaled likelihood ratio*** *(MSLR) property if, for all $\lambda \leq 1$ and x in the support of X, $f(\lambda x)/f(x)$ is monotonic in x.*

This property is satisfied by many commonly used non-negative distributions, including the gamma, Weibull, uniform, exponential, power, beta, chi, and chi-squared distributions (see the technical appendix).[1] It is related to the monotone likelihood ratio (MLR) property (Karlin and

[1] The only result dependent on this assumption is the equilibrium existence result, for which the MSLR property is a sufficient, but not necessary, condition. We have numerically observed that an equilibrium exists for many distributions that do not satisfy the MSLR assumption, such as the truncated normal.

Rubin 1956). In fact, if the distribution in question can be characterized by a scale parameter and satisfies the MLR property, then the distribution satisfies the MSLR property.

The population of consumers is divided into three distinct segments with the following characteristics (summarized in Table 1):

Bargain Hunting Consumers. Bargain hunting consumers only purchase the product when it is on sale. These consumers do not even consider the product in the full price period, possibly because they do not physically visit the retailer, because their valuation is less than p in the first period, or because they derive some utility from getting a good deal. Best Buy, for example, has famously labeled these customers as "devils," in contrast with Best Buy's favorite customers, referred to as "angels," who purchase at the full price (McWilliams 2004). In the sale period, each bargain hunter has value v_B for the item, and so her surplus from purchasing an item is $v_B - s$. There are an unlimited number of bargain hunters and they purchase, like the other segments, whenever their surplus is non-negative.[2] These consumers are analogous to the salvage market in a newsvendor formulation; as such, we make the usual assumption that $v_B < c$.

Myopic Consumers. Myopic consumers are the opposite of bargain hunters in the sense that they only purchase in the first period. The first period valuation of each myopic consumer is $v_M \geq p$, and these consumers comprise a fraction $(1-\alpha)$ of the initial (first period) demand. Thus, there are a total of $(1-\alpha)D$ myopic consumers that visit the retailer in the first period, where $\alpha \in [0, 1]$ and D is a random variable. Myopic consumers only purchase in the first period (and at the full price), either because they are unwilling to return to the retailer in the second period, because their value for the item in the second period is low (e.g., if their value is v_B or lower they clearly always prefer to purchase in the first period), or because they are simply shortsighted.

Strategic Consumers. Both myopic and bargain hunting consumers only have one decision: to purchase, or not to purchase. The final group of consumers have an additional decision: *when* to purchase. Thus, these consumers are the "strategic segment" in the sense that they are non-myopic. They consider their surplus from purchasing the product at the full price and their surplus from purchasing the product on sale, choosing between the two to maximize their expected surplus. There are αD strategic consumers and, like myopic consumers, they each have value v_M for the

[2] We have results for the comparable model with a finite number of bargain hunters. This change does not alter the qualitative results, but does complicate the analysis of equilibrium.

item in the first period. Consequently, variation in α alters the degree of sophistication in the consumer population without altering the underlying valuations or the number of consumers (i.e., there are always D consumers in the first period with value v_M, and a fraction α of these are strategic).

In the sale period, strategic consumers' values for the item are uniformly distributed in the interval $[\underline{v}, \overline{v}]$, where $\overline{v} \leq p$ and $\underline{v} \geq v_M - p + v_B$.[3] The lower bound on $\underline{v}$ is not restrictive, as any strategic consumer with value less than $v_M - p + v_B$ may be thought of as a myopic consumer (i.e., that consumer always purchases in the first period). The upper bound on $\overline{v}$ ensures that markups are never optimal and that all consumer values weakly decline over time (since $p \leq v_M$), reflective of either seasonality in the product or of discounting of future consumption.[4] We define $G(\cdot)$ and $g(\cdot)$ to be, respectively, the distribution and density functions of strategic consumer valuations, with $\overline{G}(\cdot) = 1 - G(\cdot)$.

The retailer and all consumers know the values of v_B, v_M, $\underline{v}$, $\overline{v}$ and α. Each strategic consumer has private knowledge of his or her own second period valuation at the start of the game. All strategic consumers arrive in the first period and, should they find the product in-stock, decide to either purchase the product at that time or wait for the sale, whichever gives them the highest expected surplus, which is defined to be the difference between the consumer's valuation and the purchase price.[5] If the product is not available, the consumer receives zero surplus, and we assume that consumers purchase the product if their surplus is greater than or equal to zero. The surplus to a strategic consumer when purchasing in the first period is $v_M - p$, while we denote the expected surplus from purchasing in the sale period by $\psi(v)$, where v is a strategic consumer's second period valuation.

[3] Uniform values lead to a linear demand curve in the sale period. In addition, much of the consumer behavior and multiperiod pricing literature assumes uniformly distributed valuations (e.g., Desai et al. 2007 and Lazear 1986). Although we have not established the existence of an equilibrium for more general distributions, we have observed numerically that an equilibrium exists with any increasing generalized failure rate distribution (see Lariviere 2006). Our subsequent results continue to hold analytically for more general distributions, conditional on the existence of an equilibrium.

[4] The model can also be solved with $\overline{v} \leq v_M$, so long as markups are not allowed (i.e., $s \leq p$). This case is notationally cumbersome, and so is omitted for ease of exposition.

[5] Alternatively, strategic consumers might choose a purchase period before traveling to the store; the resulting model is identical, so long as these consumers are committed to shop in one of the two periods. An interesting extension incorporates an option to not shop at all and an explicit cost to shop, as in Dana and Petruzzi (2001). In that case, first period demand can be increasing in the inventory level: by choosing a higher first period availability, consumers are more likely to shop relative to the "not shop at all" option given that shopping is costly. This effect argues for a higher initial stocking quantity, which counteracts the effect we identify for a lower initial stocking quantity.

Segment	Number	Period 1 Valuation	Period 2 Valuation
Myopic	$(1 - \alpha)D$	v_M	-
Strategic	αD	v_M	V
Bargain Hunters	*Infinity*	-	v_B

Table 1. Characteristics of the three consumer segements. $V = [\underline{v}, \overline{v}]$ is the interval of strategic consumer second period valuations.

The sequence of events is depicted in Figure 2. We model the game between the retailer and the strategic consumers as simultaneous. In other words, the retailer is incapable of credibly committing to a specific quantity (i.e., consumers do not observe the inventory level when making a purchasing decision).[6] Each player in the game (the retailer and each individual consumer) possesses beliefs about the actions of the other players. Throughout, we use the "hat" symbol ($\widehat{\cdot}$) to denote beliefs. For example, consumers believe that the stocking level of the retailer is $\widehat{q}$. When we wish to make explicit the dependence of the the retailer's profit on beliefs, we denote the profit function $\pi(q, \widehat{v})$, where $\widehat{v}$ represents the beliefs about consumer behavior. (The precise nature of $\widehat{v}$ will be discussed later.) Similarly, the second period surplus of a strategic consumer with valuation v can be written $\psi(v, \widehat{v}, \widehat{q})$, which highlights the fact that each consumer possesses beliefs about both the actions of the retailer and the actions of other consumers.

We seek to identify a rational expectations equilibrium (see Muth 1961, and operational applications by Besanko and Winston 1990 and Su and Zhang 2005). A rational expectations equilibrium ensures that the beliefs of all players are consistent with the equilibrium outcome. Hence, all consumers have the same beliefs about the retailer's behavior and the behavior of other consumers, which leads to the following preliminary result.

Lemma 1 *In a rational expectations equilibrium, there exists some $v^* \in [\underline{v}, \overline{v}]$ such that all strategic consumers with second period value less than v^* purchase in the first period, and all consumers*

[6] This does not mean that the retailer's stocking quantity has no influence on consumer behavior. Consumer behavior depends on their expectation of the retailer's stocking quantity and in equilibrium that expectation must be correct. Thus, the retailer's stocking quantity influences consumer behavior through their long run expectation. We also considered an extension of the model in which consumers observe q before making their decisions (i.e., a sequential game with the retailer moving first). This clearly works to the advantage of the retailer - by directly influencing consumer expectations the retailer can be no worse off. Nevertheless, our qualitative conclusions continue to hold.

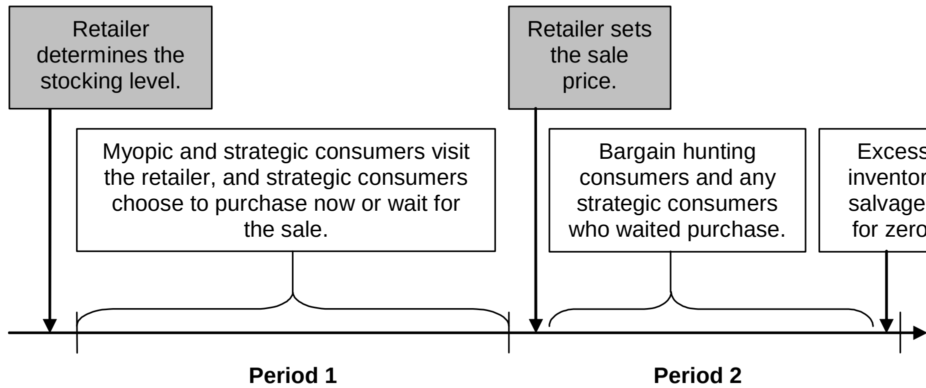

Figure 2. The sequence of events in the base model.

with value greater than v^* wait for the sale period. A consumer with value v^* is indifferent between purchasing in the first or second periods.

Proof. The proofs of all lemmas appear in the technical appendix. ■

As a result of Lemma 1, we may simplify the action space of the strategic consumers. Rather than concern ourselves with the purchasing decision of each individual consumer, we may consider instead the equilibrium value threshold $v^*(\widehat{q})$ that is induced by $\widehat{q}$. We refer to $v^*(\widehat{q})$ as the consumer best response correspondence. (Note that we do not claim now, nor is it true in general, that there is a unique best reply to a given $\widehat{q}$.) Given Lemma 1 and $v^*(\widehat{q})$, we may now define the equilibrium to this game. The following three sections proceed to characterize the RE equilibrium.

Definition 2 *A **rational expectations (RE) equilibrium** (q^*, v^*) to the game between the retailer and strategic consumers satisfies:*

1. *The retailer plays a best response given beliefs about consumer behavior: $q^* \in \arg\max_{q \geq 0} \pi(q, \widehat{v})$;*
2. *The consumers play a best response given beliefs about retailer behavior: $v^* \in v^*(\widehat{q})$;*
3. *Beliefs are consistent with the equilibrium outcome: $\widehat{q} = q^*$, $\widehat{v} = v^*$.*

4 The Retailer's Profit Function

In this section, we first derive the retailer's optimal sale price, which is chosen at the start of the second period. We then analyze the retailer's initial stocking decision, and demonstrate that the retailer's profit function is quasi-concave in q, a critical feature for proving existence of an equilibrium in §6.

4.1 Pricing in the Sale Period

As a consequence of Lemma 1, the retailer's only rational belief is that the proportion $\xi \equiv 1-\overline{G}(\widehat{v})\,\alpha$ of total first period demand attempts to purchase in the first period. (Although some consumers may be indifferent between the two periods, indifferent consumers have measure zero and their behavior is therefore inconsequential to the retailer.) The retailer's expected profit given belief $\widehat{v}$ is thus

$$\pi(q,\widehat{v}) = \mathbb{E}\left[p\min(q,\xi D) - cq + \max_{s} R(s,I)\right], \tag{1}$$

where the on-hand inventory at the start of the sale period is $I = (q-\xi D)^{+}$. Given a sale price s, consumers purchase the product if their valuation weakly exceeds s; thus, the retailer's second period revenue function is

$$R(s,I) = \begin{cases} s\min\left(\overline{G}(s)\,\alpha D, I\right) & \text{if } \overline{v} \geq s \geq \widehat{v} \\ s\min\left(\overline{G}(\widehat{v})\,\alpha D, I\right) & \text{if } \widehat{v} > s > v_B \\ sI & \text{if } s \leq v_B \end{cases}.$$

The following lemma demonstrates the form of the optimal sale period pricing policy given this revenue function.

Lemma 2 *Define the critical demand levels* $D_l = q/\left(\xi + s_m\overline{G}(s_m)\,\alpha/s_l\right)$, $D_m = q/\left(\xi + \overline{G}(s_m)\,\alpha\right)$, *and* $D_h = q/\xi$, *where* $l, m,$ *and* h *stand for low, medium, and high, respectively. Then given a demand level* D, *there is a unique optimal sale price determined by*

$$s^{*}(D) = \begin{cases} s_h(D) & \text{if } D_m < D \leq D_h \\ s_m & \text{if } D_l < D \leq D_m \\ s_l & \text{if } D \leq D_l \end{cases},$$

where $s_l = v_B$ *is the low sale price,* $s_m = \arg\max_{s \geq \widehat{v}} s(\overline{v} - s)$ *is the medium sale price, and* $s_h(D) = (\overline{v} - \underline{v})(D-q)/\alpha D + \widehat{v}$ *is the high sale price, which is contingent on the demand realization and*

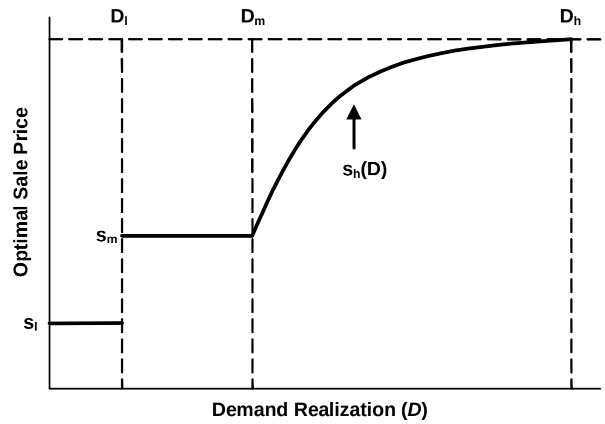

Figure 3. The optimal sale price as a function of D.

remaining inventory.

The form of the optimal policy is natural (see Figure 3). If demand is greater than D_h, the retailer sells out in the first period. If demand is between D_h and D_m, the retailer sets the highest price that clears inventory (selling only to the strategic segment). If demand is between D_m and D_l, the retailer has ample inventory and chooses the revenue maximizing price to serve the strategic segment (and some inventory is left unsold). Finally, if demand is less than D_l, there is a large amount of inventory at the start of the sale period, so the retailer prices to clear all remaining inventory with lowest sale price. In fact, if all consumers are myopic (i.e., $\alpha = 0$) then $D_l = q$, which implies the clearance price is deterministically equal to s_l in period 2.

4.2 The Initial Inventory Decision

By substituting the optimal sale price function from Lemma 2 into the retailer's profit function in (1), we may analyze the retailer's initial inventory decision. The following lemma demonstrates that the retailer's profit function is unimodal.

Lemma 3 *The retailer's profit $\pi(q, \widehat{v})$ is quasi-concave in q, and the optimal order quantity is determined by the unique solution to the first order condition,*

$$\frac{d\pi(q,\widehat{v})}{dq} = p - c - pF(D_h) + s_l F(D_l) + \int_{D_m}^{D_h} (2s_h(x) - \overline{v})\, dF(x) = 0. \tag{2}$$

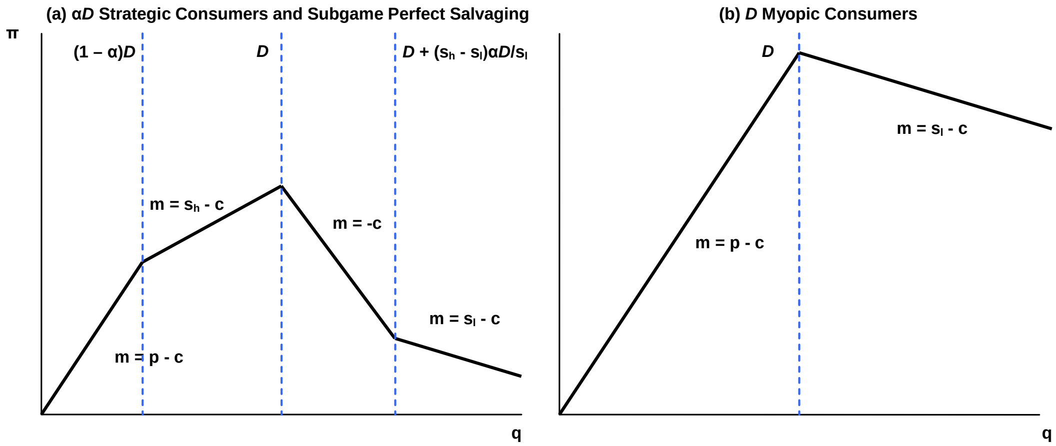

Figure 4. (a) An example of non-concavity of the retailer's profit function with deterministic demand, all strategic consumers purchasing in period two, and all strategic consumers with valuations equal to s_h (m denotes slope or margin). (b) The retailer's profit function in the same example with $\alpha = 0$.

Unlike the profit function in a traditional newsvendor model (which corresponds to our model with no strategic consumers, $\alpha = 0$), the retailer's profit function is generally not concave. To illustrate why, part (a) of Figure 4 plots the retailer's profit function in the simple case with deterministic demand, $\overline{v} = \underline{v} = s_h$ (i.e., homogeneous strategic consumers), $(1-\alpha)D$ myopic consumers and αD strategic consumers, when the retailer expects all strategic consumers to purchase in the sale period. The retailer sells the first $(1-\alpha)D$ units to the myopic consumers and the next αD units to the strategics at a lower marginal rate. Only when initial inventory is quite ample, above $D + (s_h - s_l)\alpha D/s_l$, does the retailer choose to clear at the low sales price, s_l. Thus, the retailer's profit function exhibits a concave-convex shape. Part (b) of Figure 4 plots the corresponding profit function for a newsvendor model ($\alpha = 0$). Note, relative to the maximum profit at the optimal order quantity, with strategic consumers the retailer is *less* sensitive to under ordering (i.e., the profit loss from a cautious order is less than in the traditional newsvendor model) and the retailer is *more* sensitive to over ordering (i.e., ordering too much reduces profit more quickly).

5 The Best Response of Strategic Consumers

According to Lemma 1, the strategic consumer with value $v^*(\widehat{q})$ is indifferent between purchasing in either period. This consumer's second period surplus is non-zero only if $s < v^*(\widehat{q})$, and from

Theorem 1, this only occurs if the retailer chooses the lowest sale price (i.e., $s = s_l = v_B$, which occurs when $D < D_l$). Hence, the expected surplus for the indifferent strategic consumer is

$$(v^* (\widehat{q}) - v_B) \times \Pr (D < D_l \text{ and the consumer receives a unit}) . \tag{3}$$

With the lowest sale price there are both strategic and bargain hunting consumers vying to purchase limited inventory. As a result, the probability the indifferent strategic consumer actually receives a unit in the sale period, which we call the fill rate, is not *a priori* guaranteed to be 100%. Hence, we must discuss how inventory is allocated when demand exceeds supply in the salvage period.

We introduce a new parameter $\theta \in [0, 1]$ which represents the level of optimism of the strategic segment. Suppose demand in the second period forms a queue composed of both strategic and bargain hunting consumers, of which only the first I customers are served. Strategic consumers represent every $1/\theta^{th}$ customer in the queue until there are no more strategic consumers and all remaining consumers are bargain hunters (i.e., strategic consumers are uniformly distributed among the first $(1 - \xi) D/\theta$ customers in the queue). Such a distribution of customers might occur if, for example, customers arrive according to a Poisson process; see Lariviere and Van Mieghem (2004) for an analysis of how strategic customer behavior might lead to Poisson arrivals. If $\theta = 1$, then all the strategic consumers are at the front of the queue (this is the assumption made in Su and Zhang 2005). If $\theta = 0$, then all strategic consumers are at the end of the queue; since there are an infinite number of bargain hunters, this implies strategic consumers are never served.

As a result of this supply allocation mechanism, the effective inventory available to strategic consumers in the sale period is θI, and the probability term in (3) is the second period fill rate conditional on $D < D_l$. The first part of the following lemma provides the precise value of this term.

Lemma 4 *(i) Define $\theta_c = s_l/s_m$ and let $D_\theta = \theta q/ (1 - \xi + \theta\xi)$. The probability an indifferent strategic consumer purchases and receives a unit in the sale period is*

$$\begin{array}{ll} F (D_l) & \textit{if } \theta_c \leq \theta, \\ F (D_\theta) + \int_{D_\theta}^{D_l} \frac{\theta I}{(1-\xi)x} dF (x) & \textit{otherwise.} \end{array}$$

(ii) The consumer best response $v^*(\widehat{q})$ *satisfies* $\lim_{\widehat{q}\to 0} v^*(\widehat{q}) = \overline{v}$ *and* $\lim_{\widehat{q}\to\infty} v^*(\widehat{q}) = \underline{v}$.

Lemma 4 demonstrates that if consumers are sufficiently optimistic (i.e., if θ is not too small), then θ is irrelevant: in that situation the consumer expects to receive a unit conditional that the lowest sale price is chosen. For simplicity, we assume $\theta_c \leq \theta$ for the remainder of our analysis. (Our results qualitatively hold even with $\theta_c > \theta$, but the analysis is more complex and the impact of strategic behavior is lessened–if strategic consumers expect to have a low fill rate in the sale period, then they are more likely to purchase in the first period, thereby acting more like myopic consumers.)

The second part of Lemma 4 shows that, as might be expected, if the retailer chooses a very low initial inventory, all strategic consumers purchase in the first period. If the retailer choose a very high initial inventory, all strategic consumers wait for the sale. These results are useful for demonstrating the existence of an equilibrium.

We now note the crucial role of the bargain hunting segment. Suppose there were no bargain hunting consumers but there continues to exist a strategic consumer with period 2 value $v^*(\widehat{q})$ who is indifferent between purchasing in either period. Because there are no bargain hunters, all consumers in period 2 are strategic and have value $v^*(\widehat{q})$ or higher (as per Lemma 1). The retailer's optimal period 2 price is then never less than $v^*(\widehat{q})$. It follows that the indifferent strategic consumer's period 2 surplus is zero, which means that consumer strictly prefers to purchase in period 1. Thus, we have established a contradiction–there cannot be an indifferent strategic consumer. Hence, without the possibility of a deep discount created by bargain hunters, all strategic consumers rationally purchase in period 1, i.e., they always behave as if they are myopic.

6 The Rational Expectations Equilibrium

We are now prepared to demonstrate the existence of an equilibrium. In addition, we derive a result comparing the equilibrium order quantity with strategic consumers ($\alpha > 0$) to the optimal order quantity without strategic consumers ($\alpha = 0$). In what follows, the superscript m signifies optimal values when all consumers are myopic.

Theorem 1 *A rational expectations equilibrium* (q^*, v^*) *to the game between the retailer and strategic consumers exists, and any RE equilibrium satisfies* $q^* \leq q^m$ *and* $\pi^* \leq \pi^m$.

Proof. *(i) Existence.* A rational expectations equilibrium (q^*, v^*) to the game between a retailer and strategic consumers exists if: (1) $\pi(q, \widehat{v})$ is quasi-concave in q, and (2) a solution to

$$\left.\frac{\partial \pi(q, \widehat{v})}{\partial q}\right|_{\widehat{v}=v^*(q)} = 0 \tag{4}$$

exists. Note that from Lemma 4, $\lim_{q \to 0} v^*(q) = \overline{v}$, and given $v^*(q)$,

$$\lim_{q \to 0} D_l = \lim_{q \to 0} D_m = \lim_{q \to 0} D_h = 0.$$

Similarly, $\lim_{q \to \infty} v^*(q) = \underline{v}$, and

$$\lim_{q \to \infty} D_l = \lim_{q \to \infty} D_m = \lim_{q \to \infty} D_h = \infty.$$

The expression for the high sale price, $s_h(D)$, satisfies $\lim_{q \to \infty} |s_h(D)| < \infty$ and $\lim_{q \to 0} |s_h(D)| < \infty$. By taking the limits of the first order condition evaluated at $\widehat{v} = v^*(q)$, we then see that

$$\lim_{q \to 0} \left.\frac{\partial \pi(q, \widehat{v})}{\partial q}\right|_{\widehat{v}=v^*(q)} = p - c > 0 \text{ and } \lim_{q \to \infty} \left.\frac{\partial \pi(q, \widehat{v})}{\partial q}\right|_{\widehat{v}=v^*(q)} = s_l - c < 0.$$

By the continuity of $\left.\frac{\partial \pi(q,\widehat{v})}{\partial q}\right|_{\widehat{v}=v^*(q)}$, a solution to (4) must exist, hence, combined with the results of Lemmas 2 and 3, an equilibrium must exist.

(ii) Equilibrium Comparison. Let $\pi^m(q)$ be the retailer's profit function with purely myopic consumers (i.e., with $\alpha = 0$). This is equivalent to the typical newsvendor model with salvage at price s_l, which yields:

$$\frac{\partial \pi^m(q)}{\partial q} = p - c - pF(q) + s_l F(q) = 0,$$

It follows that

$$\frac{\partial \pi^m(q)}{\partial q} - \frac{\partial \pi(q, \widehat{v})}{\partial q} = p(F(D_h) - F(q)) + s_l(F(q) - F(D_l)) + \int_{D_m}^{D_h} (2s_h(x) - \overline{v})\, dF(x). \tag{5}$$

Because $s_h(x) \geq \overline{v}/2$, each term in (5) is positive. Therefore, for any $\widehat{v}$ the optimal myopic order quantity is (weakly) greater than the optimal order quantity with strategic consumers and the optimal profits exhibit the same relationship. The result also holds for any equilibrium belief. ■

Theorem 1 demonstrates that the retailer orders *less* with strategic consumers than with myopic consumers; by creating a stockout risk, the retailer induces some strategic consumers to purchase at the full price, a result that agrees with previous findings in the consumer behavior literature (e.g., Su and Zhang 2005 and Liu and Van Ryzin 2005). We also note that while Theorem 1 proves the existence of an equilibrium, multiple equilibria to the game can exist. However, in our numerical analysis (discussed in section 9) we found multiple equilibria in only 2.5% of our sample (21 instances out of 840 cases). Thus, while multiple equilibria may occur, it appears that such cases are rare.

7 The Value of Quick Response

In this section we analyze a retailer with quick response capabilities, and explore precisely how strategic consumer behavior affects the value of a quick response system. To model quick response, we modify the base model described in §3 by allowing the retailer to submit and receive an additional order at the start of the first period after observing demand, D. The original order before the selling season remains (i.e., before observing demand D) and those units cost the retailer c_1 per unit. Units procured in the second order cost c_2 per unit, where $p \geq c_2 \geq c_1$, and they are received prior to any potential stockout (i.e., all demand in the first period is served).[7] With purely myopic consumers, this model is equivalent to the quick response with reactive capacity model in Cachon and Terwiesch (2005). As such, we use the subscript r to denote "reactive capacity" where relevant. Figure 5 depicts the new sequence of events.

Our first result mirrors Lemma 2 by providing the form of the optimal second period pricing policy with quick response.

Lemma 5 *Assume the retailer has quick response capabilities. (i) Let* $s_r = \arg\max_{s \geq \widehat{v}} (s - c_2)\overline{G}(s)$

[7] The results should remain qualitatively unchanged if excess first period demand is lost prior to receiving a replenishment via quick response, or if excess first period demand is fulfilled at the full price (possibly with some additional cost due to expedited shipping, etc). Furthermore, our results extend to the case of an imperfect demand signal.

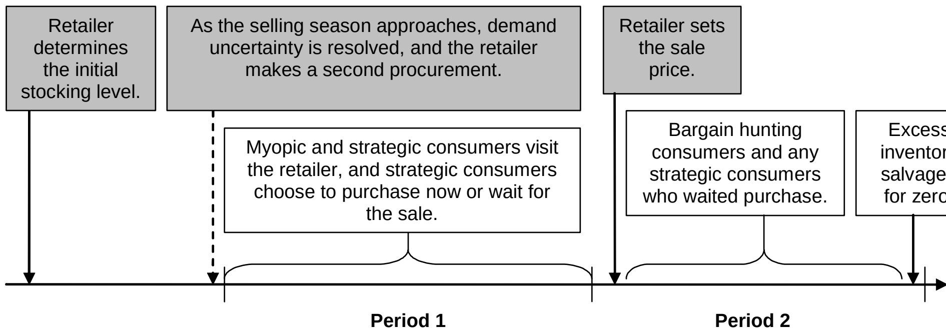

Figure 5. Sequence of events with quick response.

and let $D_r = q/\left(\xi + \overline{G}(s_r)\,\alpha\right)$. *Then, if* $c_2 \leq \overline{v}$, *given a demand level* D, *there is a unique optimal sale price determined by*

$$s^* = \begin{cases} s_r & \text{if } D_r < D \\ s_h(D) & \text{if } D_m < D \leq D_r \\ s_m & \text{if } D_l < D \leq D_m \\ s_l & \text{if } D \leq D_l \end{cases} .$$

where D_l, D_m, s_l, s_m, *and* $s_h(D)$ *are as in Lemma 2, and* $D_r \leq D_h$ *from Theorem 1. If* $c_2 > \overline{v}$, *then reactive capacity is never used to satisfy sale period demand, and the optimal sale price is identical to that derived in Lemma 2.*

(ii) The retailer's profit with quick response, $\pi_r(q, \widehat{v})$, *is quasi-concave in* q.

As a consequence of Lemma 5, the consumer best response function is identical with and without quick response. The consumer best response depends only on the probability of a low sale price (s_l), and this probability is the same for a given q with and without quick response: if the retailer's optimal action is to offer a deep discount to clear excess inventory, then quick response is clearly of no use to the retailer. (While the addition of quick response does not change $v^*(\widehat{q})$, the equilibrium in general does change.) In other words, the ability of the retailer to obtain an inventory replenishment after learning demand information does not alter consumer behavior because that capability is only put to use when demand is high and the discount in the sale period is relatively small. This is a robust result because it is never profitable to both procure additional inventory and serve the lowest value segment.

Analogous to Theorem 1, an equilibrium exists in the quick response game.[8] Furthermore, quick response induces the retailer to lower its initial stocking level and the retailer earns a higher profit with quick response than without.

Theorem 2 *A rational expectations equilibrium* (q_r^*, v_r^*) *to the game between the retailer with quick response and strategic consumers exists, yielding equilibrium expected profit* π_r^* *and satisfying* $q_r^* \leq q^*$ *and* $\pi_r^* \geq \pi^*$*. Furthermore, if*

$$\frac{v_M - p}{\overline{v} - v_B} \geq \frac{c_2 - c_1}{c_2 - v_B}, \tag{6}$$

then in equilibrium all strategic consumers purchase in the first period.

Proof. *(i) Existence.* The proof is identical to Theorem 1. *(ii) Equilibrium Comparison.* We note that the model without quick response is equivalent to the model with quick response, with $c_2 = p$. By analyzing how equilibrium quantities and profits change as a function of c_2, we may derive the results. There are subsequently three cases: either $c_2 \leq \overline{v}$ and $s_r = (\overline{v} + c_2)/2$ or $s_r \neq (\overline{v} + c_2)/2$, or $c_2 > \overline{v}$. By substituting the optimal sale period pricing policy from Lemma 5 into the retailer's profit function and taking partial derivatives, we have, for the $s_r \neq (\overline{v} + c_2)/2$ case or the $c_2 > \overline{v}$ case,

$$\frac{\partial \pi_r}{\partial c_2} = \int_{D_h}^{\infty} (q - \xi x)\, dF(x) \leq 0 \text{ and } \frac{\partial^2 \pi_r}{\partial q \partial c_2} = \overline{F}(D_h) \geq 0.$$

If $s_r = (\overline{v} + c_2)/2$, then

$$\frac{\partial \pi_r}{\partial c_2} = \int_{D_r}^{\infty} \left(q - \left(\xi + \overline{G}(s_r)\alpha\right) x\right) dF(x) \leq 0 \text{ and } \frac{\partial^2 \pi_r}{\partial q \partial c_2} = \overline{F}(D_r) \geq 0.$$

From the Implicit Function Theorem and the fact that the indifferent consumer's surplus (and hence the consumer best response) contains no explicit dependence on c_2,

$$\frac{\partial q_r^*}{\partial c_2} = -\frac{\partial^2 \pi_r}{\partial q \partial c_2} \Big/ \frac{\partial^2 \pi_r}{\partial q^2} \geq 0.$$

Thus, it follows that q_r^* is greatest when c_2 is largest, i.e., when $c_2 = p$ and there is effectively no

[8] It is important to note here that multiple equilibria may also exist in this game, just as in the game without quick response; however, in 840 numerical examples with quick response, we found no instance of multiple equilibria.

quick response option. From the Envelope Theorem,

$$\frac{d\pi_r^*}{dc_2} = \frac{\partial \pi_r}{\partial c_2} + \frac{\partial \pi_r}{\partial q}\frac{\partial q_r^*}{\partial c_2} = \frac{\partial \pi_r}{\partial c_2} \leq 0.$$

Hence, equilibrium profits are smallest when $c_2 = p$, i.e., when there is no quick response *(iii) All Consumers Purchasing Early.* If all strategic consumers purchase in the first period in equilibrium, then the equilibrium stocking level must be the myopic optimal with quick response, q_r^m, and the consumer best response must be equal to $\overline{v}$. To see when $v^*(q_r^m) = \overline{v}$, we note that this occurs when all consumers have an incentive to purchase in the first period, i.e. when

$$v_M - p \geq F(q_r^m)(\overline{v} - v_B).$$

Since $q_r^m = F^{-1}\left(\frac{c_2-c_1}{c_2-v_B}\right)$, this condition reduces to (6). ■

The final result in Theorem 2 provides a condition for when quick response induces all strategic consumers to purchase at the full price. In these cases quick response enables the retailer to restrict its initial stocking quantity to a point that effectively eliminates strategic behavior: given the retailer's low initial inventory, a strategic consumer expects only a very small probability the retailer will offer a deep discount in the second period, and thus the consumer is better off buying at the full price in period 1.[9]

The next theorem provides our main analytical result: under mild conditions, quick response is more valuable to a retailer that has strategic customers than to a retailer that has only myopic consumers.

Theorem 3 *If (6) holds, the value of quick response, given by $\Delta = \pi_r^* - \pi^*$, is greater if some consumers are strategic than if all consumers are myopic.*

Proof. Let $\Delta_m = \pi_r^m - \pi^m$ be the value of quick response with purely myopic consumers ($\alpha = 0$). If (6) holds, since all consumers purchase in the first period with quick response, $\pi_r^* = \pi_r^m$, and

[9] This result emphasizes that strategic consumers may exist in a market even if their behavior in equilibrium mirrors the behavior of myopic consumers. If the retailer were to increase its quantity (possibly based on the incorrect conjecture that the lack of strategic behavior implies a lack of strategic consumers) then the retailer may start to observe explicit strategic behavior.

hence

$$\Delta - \Delta_m = (\pi_r^* - \pi^*) - (\pi_r^m - \pi^m) = \pi^m - \pi^* \geq 0,$$

where the inequality follows from Theorem 1. ∎

Theorem 3 is concerned with the absolute increase in profit due to quick response. An immediate consequence of the theorem combined with the result of Theorem 1 is that the relative (percentage) increase in profit is also greater with strategic consumers than with myopic consumers.

Corollary 1 *If (6) holds, the percentage increase in profit due to quick response, given by* $\Delta/\pi^* = (\pi_r^* - \pi^*)/\pi^*$*, is greater if some consumers are strategic than if all consumers are myopic.*

With myopic consumers it is well known that quick response allows the retailer to better match its supply to its exogenous demand. Demand is endogenous with strategic consumers, so quick response provides the additional benefit of influencing demand. In particular, quick response allows the retailer to force strategic consumers to buy at the full price rather than wait for a possible discount: the retailer's optimal quick response quantity can be sufficiently low that all strategic consumers purchase in the first period because they expect that a deep discount is unlikely in the second period. However, it should be noted that quick response is not always more valuable in the presence of strategic consumers. Suppose strategic consumers purchase in the second period either with or without quick response. Then there are $(1 - \alpha)D$ full price consumers with $\alpha > 0$, but D full price consumers when $\alpha = 0$. In that case, quick response can be more valuable with myopic consumers because the myopic consumer case has more full price demand. Nevertheless, in §9 we find that quick response is more valuable with strategic consumers in the vast majority of situations (i.e., condition (6) is merely sufficient for the results of Theorem 3 and Corollary 1).

8 Setting the Full Price

We now let the retailer set the full price p in addition to the order quantity before the start of the initial selling season. We are interested in the optimal price path with and without strategic consumers. Consider the base model (i.e., there is no midseason replenishment opportunity) and the following dynamics: the retailer chooses the first period price p, then the retailer and consumers simultaneously choose the inventory level and the purchase period, respectively. Thus, the simultaneous game analyzed in §§4–6 is embedded in a Stackelberg game in which the retailer acts as a leader in setting the price.[10]

To ensure that the only difference between strategic and myopic consumers is their behavior, we assume their valuations are identical across the segments. In particular, like strategic consumers, myopic consumers have second period valuations uniformly distributed in the interval $[\underline{v}, \overline{v}]$ and return to the store in the second period if they do not purchase in the first period. Hence, if the retailer sets $p > v_M$, all myopic demand is shifted to the sale period, whereas if $p \leq v_M$, myopic demand occurs in the full price period.

Theorem 4 *With strategic consumers and subgame perfect salvaging, the optimal first period price (p^*) is less than or equal to v_M. With myopic consumers, the optimal first period price (p^m) is v_M.*

Proof. For the case of strategic consumers, we argue by contradiction that $p > v_M$ cannot be optimal for the retailer. With $p > v_M$ there is no demand in the first period. If the retailer sets $p = v_M$, the worst case occurs if all strategic consumers purchase in the second period and all myopic consumers in the first. Because valuations decline over time, the retailer earns more per unit on sales to myopic consumers in the first than sales to myopic consumers in the second period. Thus, $p > v_M$ cannot be optimal. With purely myopic consumers, the optimal first period price (p^m) is clearly v_M, because this is the largest price which induces the consumers to purchase in the first period. ■

Figure 6 demonstrates graphically how expected prices evolve. According to Theorem 4, with myopic consumers prices fluctuate between extremes; v_M is optimal in the first period, while v_B is optimal in the sale period. With strategic consumers the initial price is (weakly) lower than v_M, to induce some strategic consumers to purchase in the first period, and (weakly) greater than v_B in the second period, because there are second period consumers with valuations above v_B. Hence, prices are less volatile across time with strategic consumers, a result consistent with the deterministic demand model studied by Besanko and Winston (1990).

[10] In effect we are assuming that consumers immediately observe price whereas inventory is not immediately observable. Hence consumers react directly to the price set by the retailer.

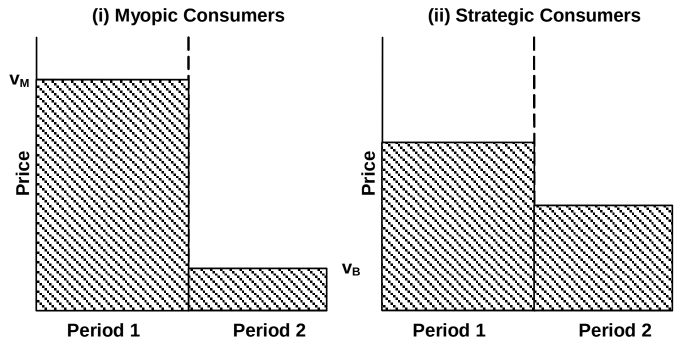

Figure 6. The evolution of expected prices over time.

9 Discussion

In this section we report on a numerical study that investigates the magnitude of the analytical results presented in the previous sections. We first constructed 1,920 examples using all combinations of the parameters in Table 7. In 1,080 out of 1,920 cases (56.25% of the initial sample) the following condition holds: $v_M - p \geq (\overline{v} - v_B)\, F\,(q^m)$, where q^m is the myopic optimal quantity. In those examples, the myopic and strategic models yield identical equilibria because all strategic consumers prefer to purchase at the full price even at the myopic optimal quantity, q^m. Consequently, it is not interesting to compare the myopic and strategic cases. Thus, we discarded those examples and restrict our attention to the remaining 840 instances in which strategic behavior occurs in equilibrium. For each of those examples we found all equilibria both with and without quick response.

Parameter	Values
Demand Distribution	Gamma
μ	100
σ	{25, 50, 100, 150}
p	10
c_1	{2.5, 5, 7.5}
c_2	$\{c_1 + 1, c_1 + 2\}$
v_M	{12, 15}
v_B	{1, 2}
V	{ [2, 10], [3, 4], [6, 7], [9, 10] }
α	{0, 0.25, 0.5, 0.75, 1}

Table 7. Parameter values used in numerical experiments. $V = [\underline{v}, \overline{v}]$ is the interval of strategic consumer values.

Table 8 presents data on the value of quick response with strategic consumers relative to the case without strategic consumers. Condition (6) holds in 76.2% of the 840 examples in the sample. Hence, in those examples Theorem 3 indicates that quick response is more valuable with strategic consumers. Among the remaining 200 examples, we find that quick response is less valuable with strategic consumers in only 11 cases. Overall, quick response is more valuable with strategic consumers than with myopic consumers in 98.7% of the 840 examples. Furthermore, the table reveals that the magnitude of the difference in value between the myopic and strategic cases can be significant. As previous work on quick response with myopic consumers has shown, the profit increase due to quick response can be enormous, quadrupling profits in some cases. We find that with strategic consumers, the potential profit increase can be far greater. In fact, if all consumers are strategic, then quick response is on average 67% more valuable than with purely myopic consumers, and can be over *five times* more valuable. Consequently, a significant portion of the value of quick response may lie in the ability of quick response to mitigate the negative consequences of strategic consumers.

Parameters		Mean Value of QR Relative to Myopic Case (Δ/Δ_m)				Maximum Value
σ/μ	c_2-c_1	α = 0.25	α = 0.50	α = 0.75	α = 1.00	of Δ/Δ_m
0.25	2	2.08	2.07	2.05	2.02	5.58
	1	1.96	1.94	1.92	1.91	5.27
0.50	2	1.74	1.87	1.88	1.89	4.71
	1	1.72	1.80	1.79	1.79	4.50
1.00	2	1.24	1.42	1.52	1.59	3.74
	1	1.29	1.46	1.54	1.59	3.51
1.50	2	1.08	1.13	1.19	1.26	2.04
	1	1.12	1.21	1.29	1.36	2.88
All		1.53	1.61	1.65	1.67	5.58

Table 8. The value of quick response (QR) with strategic consumers, Δ, relative to the value of quick response with myopic consumers, Δ_m.

According to Table 8, the relative value of quick response is highest when the cost of quick response is high (i.e., when $c_2 - c_1$ is large) and when demand variability is low. In those scenarios quick response does not add much value as a means of reacting to updated forecast information, but it does add significant value by inducing strategic consumers to purchase at the full price.

We have generally observed that the value of quick response is roughly concave in α (though it need not be monotonic). Figure 9 provides an example; for small α, the value of quick is response

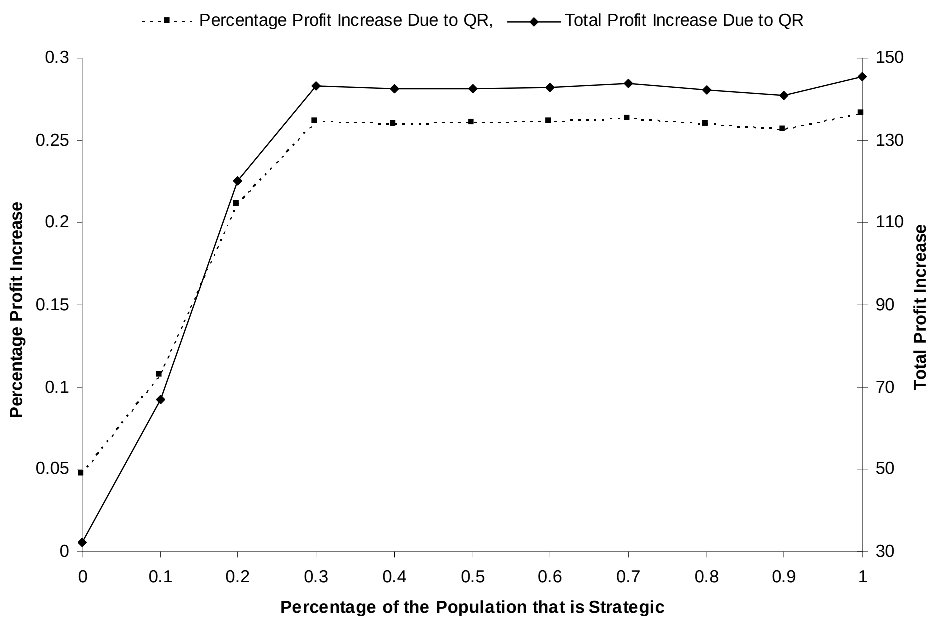

Figure 9. The value of quick response (expressed both in absolute profit increase and percentage profit increase) as a function of α, with $p = 10$, $c_1 = 2.5$, $c_2 = 3.5$, $v_M = 12$, $v_B = 2$, $[\underline{v}, \overline{v}] \in [6, 7]$, and demand gamma distributed with mean 100 and standard deviation 50.

is rapidly increasing in α, while for any α greater than 0.3, the value is relatively flat. The consequence is that a small number of strategic consumers in the population is enough to produce a rather large impact on the retailer's decisions and the value of quick response. In the entire sample of 840 examples, an average of 88% of the maximum potential profit increase due to quick response is captured when $\alpha = 0.25$.

So far we have assumed the retailer correctly anticipates the presence of strategic consumers. However, it is interesting to measure the reduction in the retailer's profit if it were to make decisions assuming all consumers are myopic when in fact they are strategic. Table 10 provides data on the cost of failing to recognize strategic behavior: if there are indeed a large number of strategic consumers, ignoring their strategic behavior can lead to a profit loss of over 90%.

We next consider when a static pricing policy is favored over a subgame perfect, dynamic pricing policy. Recall, Aviv and Pazgal (2005) find that static pricing may be preferred over dynamic pricing and Liu and van Ryzin (2005) assume the retailer commits to a static pricing policy. In our model, if the retailer chooses to commit to any particular sale price (and is able to

α	Mean % Cost of Ignoring Strategic Behavior	Max % Cost of Ignoring Strategic Behavior
0.25	2.46%	10.84%
0.50	6.41%	32.07%
0.75	10.87%	64.37%
1.00	15.87%	90.51%

Table 10. The average and maximum profit loss incurred when ignoring strategic behavior (cases with multiple equilibria excluded).

commit to a sales price), then the optimal action is to choose to not markdown at all.[11] As a result, all strategic consumers purchase in the first period, and no bargain hunters purchase in period two. Hence, price commitment is beneficial in that it shifts strategic demand to the full price period, but it is costly in that the retailer forgoes the opportunity to salvage inventory. Whether or not static pricing is a prudent strategy depends on the relative importance of those two effects.

According to Table 11, static pricing can be substantially more profitable than dynamic pricing, but it is better than dynamic pricing in fewer than 8% of cases. Table 12 presents another view of the data: sorted by the six different newsvendor critical ratios (i.e., $(p-c)/(p-s_l)$) used in our sample. As this table shows, committing to a high sale price is only profitable when the critical ratio is very high (greater than 0.8333). In these cases, margins in the first period are large and the cost of left over inventory is low. Hence, there can be considerable value in inducing all strategic consumers to purchase at the full price. Overall, despite the appeal of using static pricing to induce strategic consumers to purchase at the full price, our model suggests that a firm is generally better off using dynamic pricing even if the firm could commit to a static pricing policy.

α	% of Examples with Profitable Sale Price Commitment	Mean % Profit Increase from Price Commitment	Max % Profit Increase from Price Commitment
0.25	4.17%	2.39%	5.24%
0.50	6.77%	4.30%	11.06%
0.75	7.81%	7.74%	38.55%
1.00	7.29%	16.35%	93.62%

Table 11. The frequency and profitability of sale price commitment. Profit increase is calculated conditional on price commitment being profitable.

[11] Committing to a sale price of s_l results in the largest number of consumers waiting for the sale, in addition to the lowest average price in the sale period, so a dynamic pricing policy is clearly preferred. Conditional on committing to any price greater than s_l, the optimal action is to price as high as possible, which induces all strategic consumers to purchase in the first period.

Critical Ratio	% of Examples with Profitable Sale Price Commitment
0.2778	0.00%
0.3125	0.00%
0.5556	0.00%
0.6250	0.00%
0.8333	17.50%
0.9375	13.75%

Table 12. Frequency of profitable sale price commitment as a function of the newsvendor critical ratio, $(p-c)/(p-s_l)$.

10 Conclusion

Some consumers act strategically: they choose not only *whether* to buy a product but *when* to buy the product. They time their purchase based on their expectations of the retailer's markdown behavior as well as their own disutility from purchasing late in the season. In our model, the retailer chooses an optimal inventory and pricing policy given his expectation of consumer behavior, and each consumer chooses an optimal purchasing strategy given her expectation of the retailer's behavior and the behavior of other consumers. We demonstrate that a rational expectations equilibrium exists and we study its properties.

We find that a retailer can incur a substantial loss in profit by ignoring strategic behavior– failing to recognize strategic behavior leads the firm to order too much inventory, which makes deep discounts to clear inventory at the end of the season more likely. When consumers expect deep discounts, they are more likely to be patient and wait for a sale. Although retailers may dislike having to take markdowns, we find that a commitment to never markdown merchandise is generally not the best approach to deal with strategic consumers (even if such a commitment could be made credibly). The better approach is to be prudent with the initial inventory and then to dynamically and optimally discount.

Our main result is that quick response capabilities can be significantly more valuable to a retailer in the presence of strategic consumers relative to the case when consumers are not strategic. It has already been established in the literature that quick response can be quite valuable when consumers are myopic (i.e., non-strategic); our result indicates that even the known value of quick response may underestimate its true value. With myopic consumers, quick response gives the retailer the ability to use updated forecasts to better match supply with demand. With strategic consumers, quick response also gives the retailer the ability to mitigate the negative consequences of strategic behavior. Furthermore, this latter benefit can be substantial.

Our result with respect to quick response is important because a firm must make an investment to develop quick response capabilities. For example, the fashion apparel retailer Zara invests in localized production, fast delivery and information technology to exchange information across the firm quickly. The results of these policies at Zara have been dramatic: Ghemawat and Nueno (2003) report that Zara performs significantly better than the competition in both the number and severity of markdowns, with only 15-20% of sales at reduced prices compared to 30-40% at similar European retailers, and markdown percentages that are half the European average of 30%. We show that investments in quick response, like those made by Zara, are easier to justify when strategic consumer behavior is fully accounted for.

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Technical Appendix to "Purchasing, Pricing, and Quick Response in the Presence of Strategic Consumers"

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1 Proofs

Lemma 1 *In a rational expectations equilibrium, there exists some $v^* \in [\underline{v}, \overline{v}]$ such that all strategic consumers with second period value less than v^* purchase in the first period, and all consumers with value greater than v^* wait for the sale period. A consumer with value v^* is indifferent between purchasing in the first or second periods.*

Proof. The surplus to a strategic consumer who purchases in the first period is $v_M - p$, which is constant and independent of a consumer's second period valuation. In the second period, a strategic consumer only purchases the product if (1) the sale price is less than or equal to their second period valuation, and (2) there is inventory available to purchase. Let $\int_0^x h(s, \widehat{v}, \widehat{q})\, ds$ be a strategic consumer's belief of the probability that the sale price is less than or equal to x and the consumer receives a unit. Then second period expected surplus of a strategic consumer with period 2 valuation equal to v is

$$\psi(v, \widehat{v}, \widehat{q}) = \int_0^v (v - s)\, h(s, \widehat{v}, \widehat{q})\, ds.$$

Since $h(\cdot)$ is independent of v due to the rational expectations hypothesis, this expression is increasing in v, and hence there is a unique v^* for which $v_M - p = \psi(v^*, \widehat{v}, \widehat{q})$. All consumers with greater valuations prefer to wait for the sale, while all consumers with lower valuations prefer to purchase at the full price. ■

Lemma 2 *Define the critical demand levels $D_l = q/\left(\xi + s_m \overline{G}(s_m)\alpha/s_l\right)$, $D_m = q/\left(\xi + \overline{G}(s_m)\alpha\right)$, and $D_h = q/\xi$, where l, m, and h stand for low, medium, and high, respectively. Then given a demand level D, there is a unique optimal sale price determined by*

$$s^*(D) = \begin{cases} s_h(D) & \text{if } D_m < D \le D_h \\ s_m & \text{if } D_l < D \le D_m \\ s_l & \text{if } D \le D_l \end{cases},$$

where $s_l = v_B$ is the low sale price, $s_m = \arg\max_{s \ge \widehat{v}} s(\overline{v} - s)$ is the medium sale price, and $s_h(D) = (\overline{v} - \underline{v})(D - q)/\alpha D + \widehat{v}$ is the high sale price, which is contingent on the demand realization and remaining inventory.

Proof. First, we note that in order for the retailer to have inventory to sell in the second period, we require $D \le D_h$. The retailer then has two choices:

(i) *Pricing to serve only strategic consumers ($s > v_B$).* Any price in the range $\widehat{v} > s > v_B$ is never optimal ($s = \widehat{v}$ always yields greater profit). The optimal price conditional on $s \geq \widehat{v}$ is the solution to

$$\arg\max_{s\geq\widehat{v}} \left(s \min\left(\overline{G}(s)\,\alpha D, I\right)\right).$$

If $D \leq q$, then the retailer is demand constrained even if he serves all strategic consumers. That is, if $D \leq q$, then $\min\left(\overline{G}(s)\,\alpha D, I\right) = \overline{G}(s)\,\alpha D$ for all $s \geq \widehat{v}$. The retailer's optimization problem then becomes

$$s_m = \arg\max_{s\geq\widehat{v}} \left(s\frac{\overline{v}-s}{\overline{v}-\underline{v}}\alpha D\right).$$

Since $s(\overline{v}-s)$ is concave, there may be an interior optimum determined by the solution to the first order condition, which yields $s^* = \overline{v}/2$, if $s^* \geq \widehat{v}$; otherwise, the optimal price is on the boundary. Note that the optimal price is independent of D and α, but does depend on $\widehat{v}$ and $\overline{v}$. The optimal profit in this region is $s_m\frac{\overline{v}-s_m}{\overline{v}-\underline{v}}\alpha D$.

Now consider the case in which $q < D \leq D_h$. In this region, if the retailer sets a low sale price, he is inventory constrained, whereas if he sets a high sale price, he is demand constrained. For any demand level D, there exists some critical price $s_h(D)$, such that the retailer's revenue function is

$$R(s,I) = \begin{cases} sI & \text{if } s \leq s_h(D) \\ s\overline{G}(s)\,\alpha D & \text{otherwise} \end{cases}.$$

In particular, $s_h(D)$ is determined by solving $I = \overline{G}(s)\,\alpha D$ for s, which yields

$$s_h(D) = (\overline{v}-\underline{v})\,\frac{D-q}{\alpha D} + \widehat{v}.$$

Recall that s_m is the maximizer of $s\overline{G}(s)\,\alpha D$. Because $s\overline{G}(s)$ is concave, if $s_h(D) \leq s_m$, the optimal sale price is s_m, whereas if $s_h(D) > s_m$, the optimal sale price is $s_h(D)$. Thus, there exists some critical demand level D_m such that for $D < D_m$, it is optimal to price at s_m, and for $D > D_m$, it is optimal to price at $s_h(D)$. D_m is determined by solving $s_h(D) = s_m$ for D, which yields

$$D_m = \frac{q}{\xi + \overline{G}(s_m)\,\alpha}.$$

(ii) *Pricing to serve the bargain hunting segment ($s = v_B$).* If the retailer sets $s = s_l$, second period revenue is $s_l I$. This yields a greater profit than pricing at s_m if and only if

$$D \leq \frac{s_l q}{s_m\overline{G}(s_m)\,\alpha + s_l\xi} \equiv D_l.$$

Since s_m maximizes $s(\overline{v}-s)$ in the interval $\overline{v} \geq s \geq \widehat{v} \geq s_l$,

$$s_l\overline{G}(\widehat{v}) \leq \widehat{v}\overline{G}(\widehat{v}) \leq s_m\overline{G}(s_m),$$

which implies $D_l \leq q$. Thus, if demand is less than D_l, it is optimal to price low to clear all inventory ($s = s_l$) and serve the bargain hunters. ∎

Lemma 3 *The retailer's profit $\pi(q,\widehat{v})$ is quasi-concave in q, and the optimal order quantity is* *determined by the unique solution to the first order condition,*

$$\frac{d\pi\left(q,\widehat{v}\right)}{dq}=p-c-pF\left(D_h\right)+s_lF\left(D_l\right)+\int_{D_m}^{D_h}\left(2s_h\left(x\right)-\overline{v}\right)dF(x)=0. \tag{1}$$

Proof. The retailer's expected profit under the optimal salvage pricing policy is

$$\begin{aligned}\pi\left(q,\widehat{v}\right)=p\int_0^{D_h}\xi x dF(x)+p\int_{D_h}^{\infty}qdF(x)-cq+s_l\int_0^{D_l}\left(q-\xi x\right)dF(x)\\+s_m\int_{D_l}^{D_m}\overline{G}\left(s_m\right)\alpha x dF(x)+\int_{D_m}^{D_h}s_h\left(x\right)\left(q-\xi x\right)dF(x).\end{aligned}$$

Differentiation of this expression yields

$$\frac{d\pi\left(q,\widehat{v}\right)}{dq}=p-c-pF\left(D_h\right)+s_lF\left(D_l\right)+\int_{D_m}^{D_h}\left(s_h\left(x\right)+\frac{ds_h\left(x\right)}{dq}\left(q-\xi x\right)\right)dF(x).$$

Taking the derivative of $s_h\left(x\right)$ with respect to q, we have $ds_h\left(x\right)/dq=-\left(\overline{v}-\underline{v}\right)/\alpha x$. Then, the first derivative reduces to (1). Let $\varphi\left(q\right)=d\pi\left(q,\widehat{v}\right)/dq$. Noting $\varphi\left(0\right)=p-c>0$ and $\lim_{q\to\infty}\varphi\left(q\right)=-c+s_l<0$, it is apparent that $\pi\left(q,\widehat{v}\right)$ possesses at least one local maximum. To demonstrate quasi-concavity of $\pi\left(q,\widehat{v}\right)$, we must show that $\varphi\left(q\right)$ has a unique zero, i.e., that $\pi\left(q,\widehat{v}\right)$ possesses a single local optimum. Given the asymptotic behavior of $\varphi\left(q\right)$, a sufficient condition for this to occur is that $\varphi\left(q\right)$ itself possesses at most one local optimum. If this is the case, then $\varphi\left(q\right)$ is either quasi-concave or quasi-convex, and $\varphi'\left(q\right)$ will possess at most one interior zero. Substituting for $s_h\left(x\right)$, $\varphi'\left(q\right)=d^2\pi\left(q,\widehat{v}\right)/dq^2$ is given by

$$\begin{aligned}\varphi'\left(q\right)=\left(\overline{v}-p\right)f\left(D_h\right)\frac{dD_h}{dq}+s_lf\left(D_l\right)\frac{dD_l}{dq}-\left(2s_m-\overline{v}\right)f\left(D_m\right)\frac{dD_m}{dq}\\-2\left(\overline{v}-\underline{v}\right)\int_{D_m}^{D_h}\frac{1}{\alpha x}dF(x),\end{aligned}$$

A local optimum is achieved ($\varphi'\left(q\right)=0$) if and only if, for any q on the interior of the support of f,

$$\begin{aligned}0=\left(\overline{v}-p\right)\frac{f\left(D_h\right)}{f\left(D_m\right)}\frac{dD_h}{dq}+s_l\frac{f\left(D_l\right)}{f\left(D_m\right)}\frac{dD_l}{dq}-\left(2s_m-\overline{v}\right)\frac{dD_m}{dq}\\-2\left(\overline{v}-\underline{v}\right)\frac{1}{f\left(D_m\right)}\int_{D_m}^{D_h}\frac{1}{\alpha x}dF(x).\end{aligned} \tag{2}$$

Recall that the MSLR assumption implies $f\left(\lambda x\right)/f\left(x\right)$ is monotonic in x for all $\lambda\geq 1$. Assume that $f\left(\lambda x\right)/f\left(x\right)$ is weakly increasing in x. (The proof is identical if $f\left(\lambda x\right)/f\left(x\right)$ is weakly decreasing in x.) Since $\overline{v}\leq p$, the first term is negative and increasing in q by MSLR assumption. Similarly, the second term is positive and increasing in q by the MSLR assumption. The third term is constant, while the fourth term is negative. We will now demonstrate that the fourth term is also increasing in q by performing a change of variable. Let $yq=x$, such that $dx=qdy$, and let $\lambda_h=dD_h/dq$ and $\lambda_m=dD_m/dQ$. Then, the integral in the fourth term is equivalent to

$$\int_{D_m}^{D_h}\frac{f\left(x\right)}{xf\left(D_m\right)}dx=\int_{\lambda_m}^{\lambda_h}\frac{f\left(yq\right)}{yf\left(\lambda_mq\right)}dy.$$

Differentiating with respect to q,

$$\frac{d}{dq}\left(\int_{\lambda_m}^{\lambda_h} \frac{f(yq)}{yf(\lambda_m q)}dy\right) = \int_{\lambda_m}^{\lambda_h} \frac{dy}{y}\left(\frac{d}{dQ}\frac{f(yq)}{f(\lambda_m q)}\right) \leq 0,$$

where the inequality follows from the MSLR assumption combined with the fact that $y \geq \lambda_m$. Thus it follows that the fourth term in (2) is increasing in q. Each term on the right hand side of (2) is increasing in q, and if a solution to the equation exists, it is unique. This implies $\varphi(q)$ has at most one interior optimum, and consequently $\pi(q, \widehat{v})$ is quasi-concave in q. ■

Lemma 4 *(i) Define* $\theta_c = s_l/s_m$ *and let* $D_\theta = \theta q/(1-\xi+\theta\xi)$. *The probability an indifferent strategic consumer purchases and receives a unit in the sale period is*

$$\begin{array}{ll} F(D_l) & \text{if } \theta_c \leq \theta, \\ F(D_\theta) + \int_{D_\theta}^{D_l} \frac{\theta I}{(1-\xi)x} dF(x) & \text{otherwise.} \end{array}$$

(ii) The consumer best response $v^*(\widehat{q})$ *satisfies* $\lim_{\widehat{q}\to 0} v^*(\widehat{q}) = \overline{v}$ *and* $\lim_{\widehat{q}\to\infty} v^*(\widehat{q}) = \underline{v}$.

Proof. (i) The probability that $D < D_l$ and a strategic consumer receives a unit is

$$\int_0^{D_l} \frac{\min((1-\xi)x, \theta I)}{(1-\xi)x} dF(x).$$

A new critical demand level D_θ is determined by the $\min((1-\xi)D, \theta I)$ term,

$$(1-\xi)D_\theta = \theta(q - \xi D_\theta).$$

If $D < D_\theta$, all strategic consumers are served in the sale period. In particular, if $D_l \leq D_\theta$, then the sale price is only low when all strategic consumers are served. By comparing D_l and D_θ, we see that this occurs when $s_l/s_m \leq \theta$.

(ii) Note that $\lim_{\widehat{q}\to 0} D_l = \lim_{\widehat{q}\to 0} D_\theta = 0$. Thus, the probability term in (**??**) goes to zero as $\widehat{q}$ approaches zero. Consequently, any strategic consumers purchasing in the second period will receive zero surplus (in expectation), while purchase in the first period will yield a strictly positive surplus. Hence, all consumers purchase in the first period, and $\lim_{\widehat{q}\to 0} v^*(\widehat{q}) = \overline{v}$. Similarly, $\lim_{\widehat{q}\to\infty} D_l = \lim_{\widehat{q}\to\infty} D_\theta = \infty$, which implies the probability term in (**??**) goes to one as $\widehat{q}$ approaches infinity. This implies all strategic consumers purchase the product at the lowest sale price in period 2, and hence, since $v_M - p < \underline{v} - v_B$ by assumption, there are no indifferent consumers and all strategic type consumers wait for the sale. ■

Lemma 5 *Assume the retailer has quick response capabilities. (i) Let* $s_r = \arg\max_{s\geq\widehat{v}}(s-c_2)\overline{G}(s)$ *and let* $D_r = q/\left(\xi + \overline{G}(s_r)\alpha\right)$. *Then, if* $c_2 \leq \overline{v}$, *given a demand level* D, *there is a unique optimal sale price determined by*

$$s^* = \begin{cases} s_r & \text{if } D_r < D \\ s_h(D) & \text{if } D_m < D \leq D_r \\ s_m & \text{if } D_l < D \leq D_m \\ s_l & \text{if } D \leq D_l \end{cases}.$$

where D_l, D_m, s_l, s_m, *and* $s_h(D)$ *are as in Lemma 2, and* $D_r \leq D_h$ *from Theorem 1. If* $c_2 > \overline{v}$, *then reactive capacity is never used to satisfy sale period demand, and the optimal sale price is identical to that derived in Lemma 2.*

(ii) The retailer's profit with quick response, $\pi_r(q, \widehat{v})$, *is quasi-concave in* q.

Proof. (i) We first note that the retailer may effectively make the sale price and second procurement decisions simultaneously (although the sale price is not enacted until the start of the second period). The retailer will always procure at least enough inventory to fulfill all first period demand. Let q_2 be the additional inventory procured *above* the total first period demand. Note that if $c_2 > \overline{v}$, then $q_2 = 0$ and the retailer's sale price decision is identical to the model without quick response; hence we need only analyze the case where $c_2 \leq \overline{v}$. Then the retailer's profit function is

$$\pi_r(q, \widehat{v}) = \mathbb{E}\left[p\xi D - c_2 (\xi D - q)^+ - c_1 q + \max_{s \leq p, q_2 \geq 0} R(s, I, q_2)\right],$$

where $I = (q - \xi D)^+$. There are consequently two cases: if $I = 0$, then the initial inventory procurement is insufficient to fill any demand in the sale period. Hence, the retailer will likely wish to procure additional inventory specifically for sale in the salvage period. Alternatively, if $I > 0$, then some inventory from the initial order remains for the sale period. We will treat each case separately.

(1) $I = 0$. Any unit sold must be procured through quick response, thus $q_2 = \overline{G}(s)\alpha D$. Thus, the retailer's margin on each unit sold is $(s - c_2)$, and second period revenue as a function solely of s is $R(s) = (s - c_2)\overline{G}(s)\alpha D$, for $s \geq \widehat{v}$. The optimal sale price is thus $s_r = \arg\max_{s \in [\widehat{v}, \overline{v}]}(s - c_2)\overline{G}(s)$, and is equal to $(\overline{v} + c_2)/2$ if this value is interior to the interval $[\widehat{v}, \overline{v}]$.

(2) $I > 0$. In this case, all first period demand was satisfied without the need to procure additional inventory, and the retailer will have positive on-hand inventory at the start of the second period even if no replenishment is made. The second period revenue is

$$R(s, I, q_2) = \begin{cases} s \min\left(\overline{G}(s)\alpha D, I + q_2\right) - c_2 q_2 & \text{if } s \geq \widehat{v} \\ sI & \text{if } s \leq v_B \end{cases}.$$

If $D < q$, then the retailer is never inventory constrained in the second period, hence the revenue function is a identical to that derived in Theorem 1, and the optimal pricing scheme is also identical. On the other hand, if $D > q$, then demand exceeds the total supply, and the retailer may wish to procure additional units. Pricing to serve the bargain hunting segment is never optimal, and $q_2 = \left(\overline{G}(s)\alpha D - I\right)^+$, hence the second period revenue as a function of s is

$$R(s, I) = s\overline{G}(s)\alpha D - c_2\left(\overline{G}(s)\alpha D - I\right)^+. \tag{3}$$

Note that additional inventory is required $(\left(\overline{G}(s)\alpha D - I\right)^+ > 0)$ if

$$s \leq (\overline{v} - \underline{v})\frac{D - q}{\alpha D} + \widehat{v} = s_h(D).$$

Thus, (3) is equivalent to

$$R(s, I) = \begin{cases} s\overline{G}(s)\alpha D & \text{if } \overline{v} \geq s \geq s_h(D) \\ (s - c_2)\overline{G}(s)\alpha D + c_2 I & \text{if } s_h(D) > s \geq \widehat{v} \end{cases}.$$

This expression is a piecewise definition of two constrained concave functions. The unconstrained maximizers of these two functions are $s_m = \max(\overline{v}/2, \widehat{v})$ and s_r (defined above), respectively. This implies that if $s_m \geq s_h(D)$, the optimal sale price is s_m (just as in Theorem 1). If $s_m < s_h(D)$, the optimal sale price is $\min(s_r, s_h(D))$. Thus, by finding the demand value for which $s_r = s_h(D)$, we may find D_r, and the result follows.

(ii) The retailer's expected profit under the optimal salvage pricing policy is

$$\pi_r(q,\widehat{v}) = p\xi\mu + c_2\int_{D_r}^{\infty}(q-\xi x)\,dF(x) - c_1 q + s_l\int_0^{D_l}(q-\xi x)\,dF(x)$$
$$+ s_m\int_{D_l}^{D_m}\overline{G}(s_m)\,\alpha x dF(x) + \int_{D_m}^{D_r} s_h(x)(q-\xi x)\,dF(x) + \int_{D_r}^{\infty}(s_r - c_2)\overline{G}(s_r)\,\alpha x dF(x).$$

Differentiation of this expression yields

$$\frac{d\pi_r(q,\widehat{v})}{dq} = c_2 - c_1 - c_2F(D_h^r) + s_lF(D_l) + \int_{D_m}^{D_r}(2s_h(x) - \overline{v})\,dF(x) = 0. \quad (4)$$

Let $\varphi_r(q) = d\pi_r(q,\widehat{v})/dq$. Noting $\varphi_r(0) = c_2 - c_1 > 0$ and $\lim_{q\to\infty}\varphi_r(q) = -c_1 + s_l < 0$, it is apparent that $\pi_r(q,\widehat{v})$ possesses at least one local maximum. In the same manner as Theorem 2, by differentiating (4) we may show that there is a unique solution to $d\pi_r(q,\widehat{v})/dq = 0$ and hence the retailer's profit function is quasi-concave in q. Noting that (4) is independent of p yields the result. ■

2 Monotone Scaled Likelihood Ratio

Definition 2. A continuous, non-negative random variable X with density f satisfies the **monotone scaled likelihood ratio** (MSLR) property if, for all $\lambda \leq 1$ and x in the support of X, $f(\lambda x)/f(x)$ is monotonic in x.

Note that the property implies the following: $f(bx)/f(ax)$ is monotonic in x, for all $a \geq b \geq 0$. The following table lists several non-negative distributions that satisfy this property. We use the notation and conventions of Bagnoli and Bergstrom (2005), Tables 1 and 2.

Name	Support	Density $f(x)$	Sign of $\frac{d(f(\lambda x)/f(x))}{dx}$
Uniform	$[0,1]$	1	0
Exponential	$(0,\infty)$	$\beta e^{-\beta x}$	$+$
Power (all c)	$(0,1]$	cx^{c-1}	0
Weibull (all c)	$[0,\infty)$	$cx^{c-1}e^{-x^c}$	$+$
Gamma (all c)	$[0,\infty)$	$\frac{x^{c-1}e^{-x}}{\Gamma(c)}$	$+$
Chi-Squared (all c)	$[0,\infty)$	$\frac{x^{(c-2)/2}e^{-x/2}}{2^{c/2}\Gamma(c/2)}$	$+$
Chi (all c)	$[0,\infty)$	$\frac{x^{(c-1)/2}e^{-x^2/2}}{2^{(c-2)/2}\Gamma(c)}$	$+$
Beta ($\omega \geq 1$)	$[0,1]$	$\frac{x^{\nu-1}(1-x)^{\omega-1}}{B(\nu,\omega)}$	$-$
Beta ($\omega \leq 1$)	$[0,1]$	$\frac{x^{\nu-1}(1-x)^{\omega-1}}{B(\nu,\omega)}$	$+$

While many of the above distributions are log-concave, it is not true that the MSLR property is equivalent to log-concavity. For example, a normal distribution with a positive mean truncated to the non-negative half-space is log-concave, but does not exhibit the MSLR property over the entire support. In addition, the MSLR property is satisfied by many distributions without log-concave densities, such as the power, Weibull, gamma, chi, and chi-squared distributions for $c < 1$, and the beta distribution with $\omega < 1$. In general, if the distribution in question can be characterized by scale and location parameters and satisfies monotone likelihood ratio (MLR) property (see Karlin and Rubin 1956), then the distribution satisfies the MSLR property.

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