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by
Vivek J. Bantwal
Howard C. Kunreuther

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A CAT BOND PREMIUM PUZZLE?*

Vivek J. Bantwal**
Howard C. Kunreuther***

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** Research Associate, Wharton Risk Management and Decision Processes Center, University of Pennsylvania, Philadelphia, PA 19104.
*** Cecilia Yen Koo Professor of Decision Sciences and Public Policy and Management, The Wharton School, University of Pennsylvania, Philadelphia, PA 19104. Co-Director of the Wharton Risk Management and Decision Processes Center.

ABSTRACT

Catastrophe bonds whose payoffs are tied to the occurrence of natural disasters offer insurers the ability to hedge events that could otherwise leave them insolvent. At the same time, they offer investors a unique opportunity to enhance their portfolios with an asset that provides an attractive return that is uncorrelated with the market. Despite its attractiveness, spreads in this market remain considerably higher than the spreads for comparable speculative grade debt. This paper uses results from behavioral economics to explain the reluctance of investment managers to invest in these products. Finally, we use simulations to illustrate the attractiveness of cat bonds under a wide range of outcomes including the possible effects of model uncertainty and structural mitigation.

1. Introduction

Losses from natural hazards have increased dramatically in the past ten years so that insurers are reluctant to continue to provide protection against catastrophic risks. Prior to 1989, the insurance industry had not suffered any losses over $1 billion and were totally unprepared for losses from Andrew and Northridge[1]. Between January 1989 and October 1998, the U.S. property/casualty industry incurred an inflation-adjusted $98 billion in catastrophe losses, more than double the catastrophic losses experienced during the previous 39 years. (Insurance Services Office 1999). Figure 1 illustrates the dramatic change in the magnitude of catastrophic losses during this period.

Advances in information technology have led to the development of sophisticated hazard simulation models which allow insurers, reinsurers, and financial institutions to estimate the probability and losses from natural disasters given the portfolio of risks which an insurer and reinsurer has in place.[2] Results from these models have shown that the industry must be prepared for events that could exceed $100 billion in insured losses. (Insurance Services Office and Risk Management Solutions 1995). In fact, a repeat of the earthquake that destroyed Tokyo in 1923 could cost between $900 billion and $1.4 trillion today. (Valery 1995).

To avoid the possibility of insolvency or a significant loss of surplus, insurers have traditionally utilized reinsurance contracts as a source of protection. Reinsurance does for the insurance company what primary insurance does for the policyholder or property owner — that is, it provides a way to protect against unforeseen or extraordinary losses. For all but the largest insurance companies, reinsurance is almost a prerequisite for offering insurance against hazards where there is the potential for catastrophic damage. In a reinsurance contract, one insurance company (the reinsurer, or assuming insurer) charges a premium to indemnify another insurance company (the ceding insurer) against all or part of the loss it may sustain under its policy or policies of insurance.

While the reinsurance market is a critical source of funding for primary insurers, the magnitude of catastrophic losses makes it implausible for them to adequately finance a mega-catastrophe. Though total insurance capital was slightly over $250 billion in 1996, Cummins and Doherty (1997) find that "a closer look at the industry reveals that the capacity to bear a large catastrophic loss is actually much more limited than the aggregate statistics would suggest."

The confluence of these factors has led financial institutions to market new types of insurance linked securities such as catastrophe bonds (cat bonds) for providing

[1] Six years prior to Andrew an industry-sponsored study had been published indicating the impacts of two $7 billion hurricanes on property-casualty insurance companies. The report indicated that no hurricane of that magnitude had every occurred before that but that "storms of the dollar magnitude are now possible because of the large concentrations of property along the Gulf and Atlantic Coastlines of the United States." (AIRAC, 1986, p.1)

[2] Applied Insurance Research (AIR), EQE, and Risk Management Solutions (RMS) are leading modeling firms who are research partners in Wharton's Managing Catastrophic Risk project. See Insurance Services Office (1996) and Dong, Shah, and Wong (1996) for overviews of catastrophic risk modeling.

protection against catastrophic risks. This solution looks promising given the fact that the $26.1 trillion U.S. Capital market is more than 75 times larger than the property/casualty industry. (Insurance Services Office 1999). Thus the capital markets clearly have the potential to enhance the capacity of the insurance industry and allow them to efficiently spread risks on a broader level.

Though the market for insurance linked securities is still in its infant stages, insurers and reinsurers have successfully transferred over $2.7 billion of catastrophe risk as of May 1999 In analyzing this market, however, Penalva-Zuasti (1997) found cat bonds to be significantly more expensive than competitive reinsurance prices. Is this just a consequence of investor unfamiliarity with these securities, or does this signal some deeper issue to be resolved before catastrophe bonds can play an effective role as a risk-bearing instrument for natural hazards and perhaps be marketed at lower rates than they currently are?

This paper uses results from behavioral economics to suggest why cat bonds have not been more attractive to the investment community at current prices. In particular, we suggest that ambiguity aversion, loss aversion and uncertainty avoidance may account for the reluctance of investment managers to invest in these products. One way to encourage investment in these instruments is to show how attractive they are under a wide range of possible outcomes at current prices. We do this by simulating potential losses for cat bonds under a wide variety of hurricane scenarios for the Miami/Dade County area, the scene of Hurricane Andrew. In particular, we show that the Sharpe ratio, a standard measure for judging the performance of these bonds, is particularly attractive even under worst case scenarios. The simulations should enable investors to better understand why cat bonds are an attractive investment despite the uncertainty associated with risks from natural disaster. This understanding may lead to an increase in the demand for these instruments and result in a reduction of future prices.

The paper is organized as follows. Section 2 describes the nature of a typical cat bond and suggests why investors may not want to purchase them despite their attractive Sharpe ratios. Section 3 examines the performance of a hypothetical cat bond under a wide range of simulations with uncertainty explicitly introduced. In Section 4 we present some models to test whether the performance illustrated in Section 3 is consistent with investor behavior. Section 5 provides some concluding remarks and suggests ways that one might utilize these findings for expanding the market for cat bonds.

2. The Cat Bond Market Today

Consider the following scenario to motivate the analysis regarding the supply and demand of cat bonds based on a hypothetical insurance company who is providing coverage against losses in the Miami/Dade County, Florida area.[3] Alpha Company is an

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[3] This scenario is based on actual data on potential losses provided to the Wharton Managing Catastrophic Risk Project by Applied Insurance Research. Similar analyses for hypothetical insurance companies taking risks against earthquakes have been undertaken using data provided to the Wharton Managing Catastrophic Risk Project by EQE for Long Beach and Risk Management Solutions for Oakland. For more details on the types of analyses undertaken see Kleindorfer and Kunreuther (1999).

insurer who wants to obtain $36 million of protection against hurricane losses exceeding $42.496 million over the next year. The chances that Alpha's hurricane losses will exceed $42.496 million during the next 12 months are estimated by experts to be 1 in 100. This provides an opportunity for an institutional investor[4] to purchase a cat bond whose payoff is tied to the hurricane losses of Alpha company during this period.

To illustrate the terms of such a cat bond, we use a simple one-period binomial model as described in a recent Goldman Sachs Fixed Income Research report (Canabarro et al, 1998).[5] The investor is assumed to buy the Alpha hurricane bond at the beginning of the risk period at par ($100). At the end of the risk period (1 year in this case), the investor will receive an uncertain dollar amount, $\tilde{X}$. With probability q (.01), Alpha will incur over $42.496 million in losses from a major hurricane. This will trigger losses on the bond in which case the investor receives a stochastic recovery amount, $\tilde{R}$. These losses will be a proportional reduction of the $36 million principal obligation of the bonds. To alleviate investor concerns about moral hazard, there is a coinsurance clause whereby Alpha Company will assume 10% of any losses in the $40 million layer in excess of the $42.496 million attachment point.[6] The other 99% of the time, the investor gets back his or her principal plus LIBOR (in this case 5.9%) and an additional spread (s) (in this case 4%).[7] The interest on the bond is guaranteed even if the principal is entirely lost. Figure 2 depicts a decision tree illustrating the scenario and Table 1 summarizes the terms of the bond.

We can measure the relative value of a bond in terms of its Sharpe ratio. Here, the Sharpe ratio is defined as the ratio of the "excess return" (over the risk free rate) to the "dollar risk" i.e., the standard deviation of returns on the bond. Table 2 presents a relative value analysis of several recent cat bonds as well as comparable grades of traditional high yield debt. This analysis clearly indicates that cat bonds are much more attractive than high yield bonds in terms of their Sharpe ratios. Sensitivity analysis indicates that this superior value would hold even if the default rates on the Ba3, B1, and B2 bonds were 10% of their historical averages. In fact, Canabarro et al (1998) show

[4] Most securitizations to date have been Rule 144A private placements that are restricted primarily to Qualified Institutional Buyers (QIBs). (Moore 1998).

[5] Note a one period model ignores issues of multiple cash flows, applicable reinvestment rates, and the term structure of interest rates. Actual cat bonds, for example, often make coupon payments semi-annually.

[6] For this problem, moral hazard refers to a tendency for the insurer to write additional policies in the hurricane-prone area and spend less time and money in their auditing of losses after a disaster. It may be difficult for the investor to monitor this behavior. A coinsurance provision such as having the insurer share a large part of the losses reduces the moral hazard problem. In addition to this feature, cat bonds generally include other mechanisms designed to protect investors from asymmetric information and disincentives on the part of the insurer. For example, insurers may agree to limit the amount of new coverage they write in hazard prone areas and allow independent third party auditing of their claims. See Canabarro et al (1998) for a discussion of how recent securitizations have addressed the moral hazard and asymmetric information problems.

[7] These parameters were chosen to be consistent with actual hurricane bonds issued in 1998. LIBOR is the London interbank offered rate which is the interest rate at which major international banks lend dollars to each other. It is frequently used as a benchmark interest rate for securities. We assume that LIBOR (l) is .4% higher than the risk free rate of 5.5%.

that, under certain assumptions, the cat bonds stochastically dominate the high yield bonds.[8]

The appeal of cat bonds has been documented in several different studies. Froot et al (1995) show that cat investments over-performed domestic bonds and that the returns on cat risks are less volatile than either stocks or bonds. Litzenberger, Beaglehole, and Reynolds (1996) demonstrate that returns on cat bonds are essentially uncorrelated with the market, making them excellent tools for portfolio diversification.[9] Miller (1998) shows that non-investment grade corporate bond default rate volatility exhibits a 90% confidence interval factor of 2.7 up or down[10]. This is the same number that Major (1999) gets from estimating the uncertainty of cat bond attachment point probabilities. Are spreads in the cat bond market too high to be easily explained by standard financial theory? This question raises an interesting set of issues similar to the debate surrounding the equity premium puzzle.[11]

Possible Explanations for High Spreads

Some may argue that there really is no puzzle. For example, other structured products (CBO's, CLO's, CMO's etc.) and emerging market debt might be more appropriate assets to compare to cat bonds than traditional high yield debt.[12] Wide spreads above LIBOR in these markets may cause investors to demand even wider spreads in the cat bond market. Inferring levels of risk aversion in these markets, however, is not easily done because of the difficulty in deriving a probability distribution of losses. Emerging market bonds, for example, don't have any default statistics to examine. Penalva-Zuasti (1997) argues that the high spreads in the cat bond market can be attributed to a novelty premium and regulatory frictions. Financial theory would predict that investors would demand compensation for the lack of liquidity in the developmental stages of a new market. In fact, it might be a puzzle if a new market immediately cleared at its equilibrium price which would emerge after several years of experience with the instrument.

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[8] By definition, asset A stochastically dominates asset B if the probability of asset A's rate of return exceeding any given level is larger than or equal to that of asset B's rate of return exceeding the same level. The authors note that this statement is more sensitive to assumptions about default probabilities and recovery distributions on the high yield bonds.

[9] They find the correlations with historical stock and bond returns to be 0.058 and .105, respectively. Froot et al. also find the correlation coefficients between cat risks and other asset classes to be statistically indistinguishable from zero. Under the Capital Asset Pricing Model (CAPM) paradigm, this "zero beta" status implies that the "fair" return on cat bonds should not exceed the risk free rate.

[10] In other words, the 95th percentile value is approximately 2.7 times the median value.

[11] The equity premium puzzle refers to an empirical problem raised by Mehra and Prescott (1985) who find that the equity premium is too high to be consistent with observed consumption behavior and the risk free rate.

[12] Canabarro et al (1998) notes that cat bond prices follow a jump process while high yield bond prices have a larger diffusion component. It can be argued that emerging market bond prices can also be characterized by large sudden jumps. Investors concerned about jump risk might require a premium since they may be unable to close out their position, and thus limit their losses, before a sudden default event.

Risk Aversion Using Expected Utility Theory

Investor risk aversion based on maximizing expected utility (EU) is often used to explain the inability of frictionless benchmark asset-pricing models to explain empirical data. However, the expected rates of returns on cat bonds suggests that investors would have to be highly risk averse to **not** want to purchase these bonds. Moore (1998) finds that the coefficient of relative risk aversion (CRRA) implied by the pricing of USAA's Residential Reinsurance bond is on the order of 30.

To put this in perspective, consider an anecdote first provided by Mankiw and Zeldes (1991) and later used by Benartzi and Thaler (1995). Suppose your consumption was determined by a coin toss. If the coin came up heads, you could have a consumption of $100,000. If the result was tails, you could have a consumption of $50,000. A CRRA parameter of 30 implies that you would be indifferent between this gamble and a certain consumption of $51,209. Clearly, most people would prefer the coin toss.

Myopic Loss Aversion and Prospect Theory

In their attempt to explain the equity premium puzzle, Benartzi and Thaler (1995) point to two behavioral concepts: loss aversion and myopia. Loss aversion refers to the phenomena that investors are more sensitive to losses than gains. (Tversky and Kahneman 1991). Myopia implies that even long-term investors evaluate their portfolios frequently. The combination of these factors, which Benartzi and Thaler term myopic loss aversion, explains the discrepancy between stock returns and bond returns. Rode, Fischhoff, and Fischbeck (1999) suggest that a prospect theory weighting function "would lead (cat bond) investors to overweight an admittedly small probability of loss and thus demand a higher return." In Section 4, we develop a model to test these hypotheses in the context of cat bonds.

Ambiguity Aversion and Comparative Ignorance

Ellsberg (1961) argues that people's willingness to act in the presence of uncertainty depends not only on the perceived probability of the event in question, but also on its vagueness or ambiguity. Fox and Tversky (1995) show in several experiments that "when people compare two events about which they have different levels of knowledge, the contrast makes the less familiar bet less attractive or the more familiar bet more attractive." This phenomena is referred to as the comparative ignorance hypothesis. Sarin and Webber (1993) show that even the market setting is not enough to eliminate this effect.

There is evidence from studies of the insurance and reinsurance industry that underwriters will charge a much higher premium for which the premiums are ambiguous and/or the losses uncertain. For example, Kunreuther et al (1995) conducted a survey of 896 underwriters in 190 randomly chosen insurance companies to determine what premiums would be required to insure a factory against property damage from a severe earthquake. as a function of the degree of ambiguity in the probability and/or uncertainty in the loss. For the case where the probability was ambiguous and the loss uncertain, the premiums were between 1.43 to 1.77 times higher than if underwriters were asked to price a non-ambiguous risk.

Investors may behave in the same manner as underwriters in that they will demand a higher spread for a bond where there is considerable ambiguity associated with the risk. In the case of natural hazards, there is considerable uncertainty surrounding the modeling of catastrophic risks. Furthermore, insurance-linked securities represent a new asset class for investors. Rode, Fischhoff, Fischbeck (1999) point out that this new asset class does not fit into the typical class of products with which investors are comfortable; in other words, cat bonds are neither equity nor debt but exhibit some characteristics of each of these standard classifications.

Selection Bias and Threshold Behavior

Value At Risk (VAR) has become the industry's standard risk management approach. (Basak and Shapiro 1999). VAR is a point estimate of the loss that will be exceeded with a prespecified probability (p) over a t day holding period Suppose that p = 1%, t = 1 day, and VAR is calculated to be $1 million given a probability distribution of losses. This implies that the threshold level is 1 - p or 99%. This would mean that in the next 100 days we would only expect to see one day where the trading losses exceed $1 million. An increase in the threshold level (e.g. from 95% to 99%) implies that an individual is more risk averse.[13] If cat bonds have a 1% risk of default, then those investors who are considering purchasing this bond would be more risk averse than investors buying bonds with a 5% default rate. This conjecture is difficult to test because information about the investors in this market is not publicly available.

Institutional arrangements can also hinder an investment manager's ability or desire to invest in cat bonds. Rode, Fischhoff, and Fischbeck point out that "Many investment managers (and mutual funds in particular) are restricted (often by charter) from purchasing securities that fall outside these two classes (debt and equity)." These restrictions help reduce market depth and liquidity by limiting involvement of the investment community. Even when investment in cat bonds is not explicitly restricted, investment managers may fear the repercussions on their reputation of losing money by investing in a "weird" asset. Unlike investments in traditional high yield debt, money invested in cat bonds can disappear almost instantly and with little warning. This potential for a sudden, large loss of capital can worry investors despite the low probability of such events occurring.

Impact of Worry

Investors may be reluctant to commit funds to new financial instruments if they spend time worrying about the possibility of losing their principal due to a catastrophic disaster. Events which have catastrophic potential and create dread are perceived by

[13] The importance of threshold levels in financial decision-making is illustrated in a recent study by Degeorge, Patel, and Zeckhauser (1999).

individuals to be very risky even though the statistical data on probabilities and consequences do not suggest that people should feel that way about them. (Slovic 1987). Catastrophe bonds may generate these concerns on the part of investors. The cost of thinking about the potential consequences of a low probability event may lead the investor to ignore the potentially high gain because the specter of losing the entire principal of the bond looms very large.

Fixed Cost of Education

The initial cost necessary to understand the legal and technical nuances of a new market may outweigh the marginal benefit of cat bonds over more familiar investments. The importance of this factor in limiting the marketing of these new financial instruments can be tested by solving for the fixed cost of education necessary to make the investor indifferent between the cat bond and a comparable bond. To the extent that issues feature similar characteristics, the cost of educating oneself about future cat bonds will shrink over time. This cost is not the same as the one associated with worry since it should decrease as learns more about the bond. One way to contrast the two would be to see if the rate of returns increased as the length of time (T) the bond was in force increased. If this is the case, it would support a worry theory. If the required return was independent of time T then this would support the Fixed Cost of Education theory. Both factors may play a role.

3. Can Cat Bonds Be Made More Attractive to Investors by Examining the Impact of Uncertainty?

One way to make cat bonds financially more attractive is to show how robust they are under a wide variety of realistic disaster scenarios. If the returns on the investment remains high even for worst case scenarios, then some of the concerns with ambiguity and uncertainty should be allayed. In this section we show the robustness of cat bonds for a wide variety of different scenarios in a Model City that is subject to possible damage from hurricanes.[14] More specifically, we analyze the performance of cat bonds (using Sharpe ratios as a guide) under a wide band of uncertainty regarding the probability of certain events occurring as well as the magnitude of the losses.

We begin by creating a hypothetical insurance company which provides coverage to residential property owners in the Miami/Dade County area which is subject to hurricanes. All residents would like to purchase insurance against wind damage from hurricanes and other storms, but not everyone can obtain coverage. None of the homes have adopted mitigation measures. Since the insurer is concerned with the possibility of insolvency, it may limit the amount of coverage it provides and some property owners may be unprotected. Table 3 shows the composition of the insurer's book of business in the Model City area

[14] Our model city is based on data for Miami/Dade County which was provided to us by Applied Insurance Research (AIR).

To estimate the company's loss potential from hurricanes, we use CLASIC™ Version 1.8. simulation software developed by Applied Insurance Research (AIR). The AIR hurricane model performs a Monte Carlo simulation that draws upon extensive historical meteorological databases to generate thousands of hypothetical storms.[15] The losses from these storms can be stochastically summed to yield a loss distribution (F(L) = Pr{Loss ≤ L}) and the associated exceedance probability (EP) function (EP(L) = Pr{Loss ≥ L) = 1 - F(L)). The resulting EP curve is a function of the hurricane events, number and type of properties and their location relative to the hurricane events, as well as the insurance and reinsurance parameters.

Despite scientific advances in the modeling of hurricanes there is still considerable uncertainty on the estimates of the probabilities associated with these events.[16] Since the presence of uncertainty is clearly an important factor in any investment decision, we see how uncertainty in the AIR model can affect the valuation of a cat bond.

As a reference point for dealing with uncertainty we construct a base case scenario (B). This scenario, depicted graphically in Figure 3, represents the experts' mean estimates for all the parameters in the Monte Carlo simulation of hurricanes. Two parameters are varied: hurricane filling rates (F) and vulnerability (V) to create high (H) and low (L) estimates relative to the base case. The values of H and L are determined so as to yield a 90% confidence interval. This means that high and low estimates will cover the true estimate of the model parameter(s) with probability .90.[17]

We define a 90% confidence interval to be one where there is a 5% chance that the damage is below L and a 5% chance that the damage is above H. In order to create such a confidence interval with respect to both parameters we proceed as follows. The 5% level of F and V is a pair of values of the relevant parameters, call them (f05, v05), such that there is only a 5% chance that the damage associated with the true value of both parameters will be less than (f05, v05). Assuming that F and V are independently distributed, the required joint probability is[18]:

(1) $\Pr\{F < f05 \text{ and } V < v05\} = \Pr\{F < f05\} \times \Pr\{V < v05\} = 0.05$

[15] See Kelley and Zeng (1996) for a complete discussion of hurricane modeling.
[16] Major (1999) discusses uncertainty in catastrophe models in detail.
[17] H and L refer to the level of losses, not to the actual hurricane filling rates or vulnerability relationships. Hurricane filling rates (F) refer to the rate at which wind speeds dissipate after a hurricane makes landfall. Vulnerability (V) relationships estimate the damage done to buildings as a result of the hurricane. These two parameters and the assumption of independence were based on discussions with the Technical Advisory Committee (TAC) of the Wharton Managing Catastrophic Risk Project. The TAC is comprised of independent engineers, physical and social scientific experts in various aspects of catastrophe modeling.
[18] This methodology is more appropriate for constructing joint 90% curves with respect to uncertainty in frequency and vulnerability, which are statistically independent. Unfortunately, this assumption does not hold for hurricane filling rates and vulnerability. AIR estimates that this violation results in a confidence interval on the order of 75 - 80% rather than 90%. See Grossi, Kleindorfer, and Kunreuther (1999) for a discussion of uncertainty in earthquake modeling.

There are, of course, an infinite number of ways to pick f05 and v05 to make this equality true. We decided to pick f05 and v05 so that, roughly, the same marginal probability would hold in (1). This means that f05 and v05 are arbitrarily set so that

(2) $\Pr\{F < f05\} = \Pr\{V < v05\} = 0.2236$

The same logic applies to determining the 95% level.

The EP functions under these different states of the world provide the foundation for evaluating the decisions made by insurers and investors. These EP curves are depicted in Figure 4.

We assume that the insurer will purchase reinsurance coverage as their first layer of protection and then rely on cat bonds for the next layer. Though the reinsurance market today is not as capacity constrained as it has been in the past, an insurer may still choose to restrict reinsurance coverage for a number of reasons. Reinsurance prices, even in a "soft market", can still be relatively expensive at high levels of retention and there is still a limit to capacity for any one cedant, particularly in high-risk areas like Miami, FL. Furthermore, the insurer may be concerned about credit risk due to the possible insolvency of the reinsurer after extreme events. For these reasons the insurer is assumed to issue a hurricane bond as a substitute for reinsurance at high layers. The terms of this hypothetical hurricane bond were detailed in Table 1.

Figure 5 shows the impact of uncertainty on Alpha Company's expected losses when only F is varied, when only V is varied and when both F and V are varied. While losses for the high curve are only 10% greater than losses for the low curve when only the filling rates (F) are varied, differences between the high and low curves in the other two scenarios are on the order of 50%. This indicates that the impact of uncertainty with respect to F is relatively minor in relation to the uncertainty that results from varying vulnerability (V). Of course, these differences relate to losses in an expected value sense. Catastrophe security pricing is more concerned with losses that occur in the right tail of the distribution.

Table 4 illustrates the impact of uncertainty on the chances that one will require using the cat bond to pay for some of the insured losses experienced by the Alpha Company. While the probability of exceeding the $42.496 million attachment point is 1% under the base case, this probability drops to .71% for the low curve and is 1.34% for the higher curve. Wide confidence intervals are not surprising in the field of catastrophe modeling. Grossi, Kleindorfer, and Kunreuther (1999) use the RMS and EQE models to perform a similar two-parameter uncertainty analysis with respect to earthquake modeling and find that the expected losses from the high curves are more than triple expected losses from the low curves. Major (1999) makes assumptions about four sources of uncertainty in hurricane models (sampling error, model specification error, nonsampling error, and process risk) and finds 90% confidence factors in the range of 3.4 to 4 for dollars and 2.7 for probabilities.

Implied Risk Aversion and The Impact of Uncertainty on the Value of a Cat Bond

How does this uncertainty affect the relative value of cat bonds at current prices? Table 5 shows the relative value of the hypothetical cat bond in the presence of uncertainty. For the current spread of s= 4%, the expected rates of return range from 9.14% (high curve) to 9.59% (low curve). Even with a high curve the Sharpe ratio is 0.46, which is considerably higher than any of the comparative speculative grade bonds depicted in Table 2 although there is greater comparability with the more recent spreads. Tables 6 and 7 show that the Sharpe ratios and expected rates of return remain high even as we reduce the spreads to half their current level. Thus when s=2%, the Sharpe ratio is still at 0.21 for a high curve.

What do these spreads imply about the risk aversion of investors? Different investor decision rules and utility functions can be used to evaluate this issue. In financial economics, it is common to assume that agents follow a time-separable power utility function, so that

$$U(W) = \frac{W^{1-\gamma} - 1}{1-\gamma}$$

where γ is the coefficient of relative risk aversion[19]. As γ approaches one, this utility function approaches the log utility function

$$U(W) = \log(W)$$

We assume the agent invests 10% of his initial wealth in these bonds.[20] Using the certainty equivalence method, we elicit values for the spread s such that the agent is indifferent between the bond and the risk free rate for given values of gamma. This amounts to solving the following equation for s

$$\text{(3)} \qquad E\left\{\frac{W^{1-\gamma}(.9(1+r)+.1\tilde{X})^{1-\gamma} - 1}{1-\gamma}\right\} = \frac{(W(1+r))^{1-\gamma} - 1}{1-\gamma}$$

where W represents the investor's initial wealth (90% of which is put in risk free securities and 10% in cat bonds), and where $\tilde{X}$ is computed from Figure 2.[21]

[19] The power utility function allows us to aggregate in a complete market setting all agents into a single representative investor with the same utility function as the individuals regardless of their wealth levels. For a complete discussion of power utility function and its important properties consult Campbell, Lo, and MacKinlay (1997).

[20] We varied the fraction of wealth invested in cat bonds to see how sensitive our results were to alternative specifications. While the fraction chosen does affect the absolute spreads demanded on the various bonds, the relationship between these spreads is not affected and therefore neither are our conclusions.

[21] The reader should note that (3), while simple in structure, is quite complicated to compute in practice. Doing so requires complete knowledge of the loss distribution for $\tilde{L}$ and of the recovery distribution $\tilde{R}$. In practice, therefore, computing the distribution of returns of monies invested in the cat bond (in this case B = $.1W\tilde{X}$) requires simulation. This is the main source of complexity referred to throughout this paper.

Table 8 shows that a risk neutral investor would demand a spread of 52 basis points under the base case scenario. The required spreads gradually widen for moderate levels of risk aversion, increasing more dramatically as the agent becomes very risk averse. The implied coefficient of relative risk aversion for our hypothetical cat bond using the base curve is on the order of 40.[22] This represents a 348 basis point risk premium on an expected loss of 52 basis points. Notice for the low curve, even extremely risk averse investors would demand spreads well below the market clearing spread of 400 basis points. For the high curve, however, the expected loss is 24 basis points higher than the base case. This 24 basis point spread can be interpreted as an uncertainty premium that investors would demand in addition to the risk premium. This implies that lower levels of risk aversion are needed to explain the market clearing spread when the high curve is used. In this case, the implied coefficient of relative risk aversion for an investor to be indifferent between a cat bond with a spread of 400 basis points (i.e. s=4%) and a risk free investment is still relatively large---approximately 33 for this example.[23]

Implied Risk Aversion in the High Yield Market

How does the implied risk aversion in the cat bond market compare to the implied risk aversion in the traditional high yield market? To answer this question, we extend the relative value analysis of Canabarro et al (1998) discussed in Section 2. Using equation (3), we calculate the spreads required for different levels of risk aversion for the base case scenario.[24] The high yield default probabilities and recovery value parameters are taken from Moody's Investor Service (1998). Recovery values are assumed to follow a beta distribution.[25] Like Canabarro et al, we consider two sets of spreads. These correspond to market conditions before the summer of 1998 (low spread) and after the summer of 1998 (high spread).

Table 9 shows the results of this analysis. Aside from the Ba2, spreads for comparable high yield bonds are consistent with modest to moderate levels of risk aversion- i.e. below 20 even for high spreads. Canabarro et al note that the short time frame used to compute the default probabilities, though indicative of the current regime, may result in statistical fluctuations from the long-term average. For example, investors may believe that B2 default rates are lower than 6.70%. This would explain why these

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[22] The cat bond we are looking at has a spread of 400 basis points (i.e. s=4%). Interpolating between constant relative risk aversion coefficients of 40 (which implies a 376 basis point spread) and 45 (which implies a 505 basis point spread), the implied risk aversion for 400 is approximately 41.

[23] In this case a relative risk aversion coefficient of 30 implies a 336 basis point spread and 35 implies a 450 basis point spread. Interpolating between these 2 values for a 400 basis point spread implies a risk aversion coefficient of 32.8.

[24] In this case, $\tilde{X}$ is the uncertain dollar return on the high yield bond rather than the cat bond.

[25] Moody's defines recovery values as the percentage of par value returned to the bondholders. This definition is justified since, in a bankruptcy proceeding, the investor has a claim on the principal but not the coupon. Note, however, that a one period model does not account for the possibility of a quarterly or semiannual coupon payment before the default event. Thus our model slightly underestimates the recovery on the high yield bonds.

investors appear to be risk-seeking in our model[26]. An alternative explanation is that investors are willing to pay an "insurance" premium for bonds that they feel are attractive from a portfolio hedging standpoint. For example, an investor may need to offset a risk with a B3 bond. In this context, the investor is concerned about the performance of the entire portfolio (i.e. existing assets plus the B3 hedging instrument) and may not be interested in the performance of the hedging instrument by itself. In any event, the levels of risk aversion in the high yield market are not consistent with observed behavior in the cat bond market.

4. Explaining the Puzzle

We now revisit some of the rationale for high cat bond prices in Section 2 and test whether any of them can explain the puzzle.

Fixed Cost of Education

As with any new market, agents must invest time and money up front in order to educate themselves about the legal and technical complexities of the cat bond market. This initial sunk cost is necessary before the investor can even make a decision on whether or not to purchase the bond. Such a transaction cost will diminish the attractiveness of the new bond, perhaps to the point where the investor would prefer to stay out of the market. Using the certainty equivalence method, we solve for the fixed cost that would leave investors indifferent between the cat bond and the comparable high yield bonds for the base case scenario. Table 10 presents the results of this analysis. For example, an investor with a risk aversion coefficient of 10 would be indifferent between the cat bond and a B1 bond if he thought that the cost of educating himself about the cat bond was $929,106.

The fixed costs will obviously vary with the level of risk aversion. In particular, we see that, with the exception of the Ba2 bond, the fixed cost necessary to equate the two markets increases as the investor becomes more risk averse. This result is not surprising. Notice that all of these bonds have higher default probabilities and higher expected losses than the cat bond. As an investor becomes more and more risk averse, he will want to avoid these riskier securities. Thus the fixed cost subtracted from the "safer" cat bond will become very large in order to leave the investor indifferent between the two bonds.

The reason we see the opposite effect with the Ba2 bonds is that it has a lower default probability and lower expected loss than the cat bond. In the risk neutral case, solving for the fixed cost simply amounts to subtracting the expected value of the two bonds. The cat bond, which offers a greater spread than the Ba2 bond, yields an extra $403,143 in expected value. As an investor becomes more risk averse, the amount subtracted from the cat bond will decrease since the investor will begin to prefer the safer asset despite the large spread of the cat bond. Interestingly, we see that a sufficiently risk

[26] In other words, this would explain why the market spread is lower than the spread that our risk neutral investor would demand.

averse investor (i.e. with a risk aversion coefficient above 36) would actually prefer the Ba2 outright, despite the cat bond's higher Sharpe ratio.

Aside from the Ba2 results, we would expect that the actual cost of educating oneself about this market is substantially less than most of the figures in this table. Nevertheless, these results underscore an important point. There is a clear disadvantage to complex structures not only because of the cognitive limitations that hinder investor decision making (Rode, Fischhoff, and Fischbeck 1999), but also because the cost of educating oneself about the bond diminishes its financial attractiveness. Furthermore, these results show that it is in the industry's best interest to standardize cat bond terms as much as possible so that the investor's fixed cost of education is only spent once.

Myopic Loss Aversion and Rank Dependent Expected Utility Theory

Economic analysis of decision making under uncertainty has been dominated by expected utility (EU) theory. In recent years, however, there has been considerable empirical evidence suggesting that EU based models lack descriptive power despite its normative rigor.[27] Allais (1950) and Ellsberg (1961) illustrated how most people violate EU's independence axiom. Tversky (1969) shows that people violate the transitivity axiom. Kahneman and Tversky (1979) demonstrate that individuals tend to be risk averse in gain situations and risk prone in loss situations, an asymmetry they term the "reflection effect." They also suggest that there is a tendency for individuals to overweight relatively low probabilities. These inconsistencies have motivated the need for a new class of models based on generalized utility theory.

We now apply one of these models to see if it better explains behavior in the cat bond market. Specifically, we utilize the rank-dependent expected utility (RDEU) model developed by Quiggin (1982). This model has some important properties that make it more consistent with observed behavioral patterns. For example, the model permits nonlinear weighting of cumulative probabilities which enables us to account for people's tendency to overweight small probabilities. Rank dependence refers to the notion that the expected utility associated with one branch of a lottery depends on how it ranks relative to other branches. In other words, one branch can overshadow or "intimidate" another branch in RDEU theory whereas each branch contributes additively to overall expected utility in EU theory. The point here is that the overweighting of small probabilities should only apply to extreme events as opposed to intermediate outcomes. This feature is necessary to explain the inconsistencies raised by Allais and others.

The RDEU function (Quiggin 1982) is given by

$$V(\{x;p\}) = \sum_{i=1}^{N} U(x_i)\, w_i(p)$$

[27] See Kleindorfer, Kunreuther, and Schoemaker (1993) for a survey of this literature.

where p is the cdf of a random variable X taking values $x_1 \leq x_2 \leq \ldots \leq x_n$ with probabilities $p_1, p_2, \ldots, p_n$ and where

$$w_i(p) = h(\sum_{j=1}^{i} p_j) - h(\sum_{j=1}^{i-1} p_j) = h(F(x_i)) - h(F(x_{i-1}))$$

with the weighting function h satisfying h' > 0, h(0) = 0, and h(1) = 1. The particular "h" used here is discussed below.

Following Kahneman and Tversky (1979), we define the utility of returns separately over gains and losses

$$U(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x < 0 \end{cases}$$

where λ is the coefficient of loss aversion which they estimate to be 2.25. They estimate α and β to be .88. X is the return on the bond implying that our reference point is the amount of the investment, \$36 million.

Tversky and Kahneman (1992) have also suggested the following one-parameter probability weighting function

$$h(z) = \frac{z^{\zeta}}{(z^{\zeta} + (1-z)^{\zeta})^{1-\zeta}}$$

where ζ is estimated to be .61 in the gain domain and .69 in the loss domain.[28]

Benartzi and Thaler (1995) apply a similar model to the equity premium puzzle. They find that the observed reluctance of investors to hold stocks can be attributed to a combination of loss aversion and a short evaluation period. Specifically, they find that investors will prefer the safety of risk free bonds over equities if their time horizon is one year or less. While it cannot be proven that a specific explanation is correct, they conclude that the myopic loss aversion hypothesis is a plausible explanation for excess premiums in the equity market.

Table 11 presents the results of our RDEU analysis. As expected, the model favors the risk free investment over all the risky assets. We also find that, using this model, the investor is almost indifferent between the cat bond and the Ba2 bond. In fact, if the cost of education associated with the cat bond exceeded \$43,500, then the investor would prefer the high yield bond. We also find that the investor would clearly prefer either the cat bond or the Ba2 bond to the other speculative grade instruments. In fact, an investor would never invest in the riskier B1, B2, and B3 bonds under this model.

[28] Note that in standard RDEU theory ζ is constant across gains and losses.

The fact that investors in the high yield market only have a claim on their principal in the event of a default helps explain this result. The model overweights the cumulative probability of default as well as the severity of the default for both types of bonds. This yields a set of negative utilities whose sum must then be offset by the positive utilities of the no default states. However, since the return on the high yield bonds is expressed as a fraction of the principal, even 100% recovery does not exceed the reference point of $36 million. The offsetting positive utility is derived from the lone event that exceeds the reference point- the no default case whose probability has been underweighted. In the case of our hypothetical cat bond, the coupon is guaranteed even if the principal is entirely lost. This yields a set of intermediate positive utility outcomes resulting from hurricanes that exceed the attachment point but not by more than the coupon payment.

Of course the main reason for the unattractiveness of these speculative grade bonds under this model is the relatively high level of default probabilities and expected losses relative to the cat bond and the Ba2 bond. Thus the RDEU model explains the puzzling excess spreads on the cat bond relative to the risk free rate, though it does not account for the wide spread differential between cat bonds and the comparable high yield bonds. We also see that incorporating reasonable fixed costs of education into the model will leave the investor indifferent between the cat bond and less speculative grade bonds.

Ambiguity Aversion and Comparative Ignorance

Fox and Tversky (1995) show that this ambiguity aversion translates into statistically significant pricing discrepancies between similar types of bets. In a series of experiments, they find that people are willing to pay considerably more for familiar bets than unfamiliar bets in the same setting. However, when considered in isolation, the price of the clear bet and vague bet are statistically indistinguishable from each other. They propose that "people's confidence is undermined when they contrast their limited knowledge about an event with their superior knowledge about another event, or when they compare themselves with more knowledgeable individuals." If true, this idea can easily be extended to the cat bond market where institutional investors might compare their limited knowledge of catastrophic risk modeling to their (perceived) expertise in the high yield market or with the superior knowledge of the insurers ceding the risk (the asymmetric information problem.)

These ideas are illustrated by looking at the behavior of insurance underwriters who demand a much higher premium if they are uncertain or ambiguous about the catastrophic risk as shown in Table 12. (Kunreuther et al 1995). What spread would investors demand on a cat bond if they exhibited these same preferences as underwriters for clear bets over vague bets? Table 13 indicates that, in many cases, the investors would demand premiums in excess of the current market spread of 400 basis points.[29] Columns 1 (Ap, L) and 2 (p, UL) show the returns that investors would demand on cat bonds if they were ambiguous about either probabilities or losses, respectively, for a $10

[29] This analysis assumes that the investor is only looking at the expected return and does not consider the variance of returns. The cat bond has a lower standard deviation of return than the other bonds (except the Ba2) but investors here assume that the risks are equivalent because of ambiguity.

million bet on a 1% probability of default. The ambiguity aversion becomes especially apparent when investors are unclear about both the probabilities and losses. The column (Ap, UL) shows the returns that investors would demand when they are ambiguous about both probabilities and losses. For example, these investors would demand 1.43 times the expected return they could earn on comparable instruments. Thus they would require spreads of 5.91%, 4.60%, 4.80%, and 2.91% above LIBOR on the cat bond when comparing them against the Ba2, Ba3, B1, and B2 bonds, respectively. The negative required spreads in the B3 section indicate that investors would prefer the cat bond despite the ambiguity. Nevertheless, this experimental evidence is consistent with observed behavior in the cat bond market. That is not to say, however, that such ambiguity aversion is justified, particularly in light of our results in Section 3.

5. Conclusions

The appeal of cat bonds has been well documented and is further confirmed with this analysis of Miami/Dade County. With high spreads that are uncorrelated with the market, these new financial instruments offer investors a unique opportunity to enhance their portfolios. In fact, spreads in this market are too high to be explained by standard financial theory, giving rise to another asset pricing puzzle which cannot be fully explained by investor risk aversion. This paper suggests that the high spreads are not just a consequence of investor unfamiliarity with a new asset but instead signal some deeper issues that need to be resolved before the cat bond market can fully develop. In particular, we show that ambiguity aversion, myopic loss aversion, and fixed costs of education can account for the reluctance of institutional investors to enter this market. Worry as to the impact of a catastrophic loss on the performance of the cat bonds may be an additional factor to consider.

Investors will be able to overcome these obstacles only after they are comfortable with both the complexity and uncertainty of the cat bond market. Issuers can address the former by standardizing a simple structure of terms so that the investor's fixed cost of education on their first cat bond will not require them to incur additional high costs when evaluating future issues. Quantifying and reducing pricing uncertainty can help investors overcome their aversion to ambiguity. Our simulations should enable investors to better understand why cat bonds are an attractive investment despite the uncertainty associated with risks from natural disaster. This understanding may lead to an increase in the demand for these instruments and result in a reduction of future prices.

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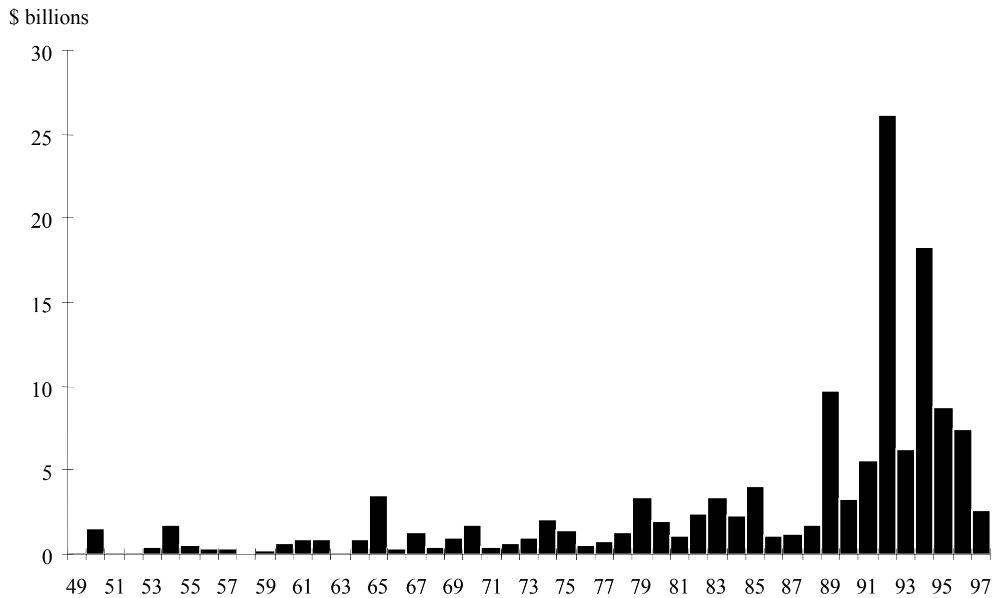
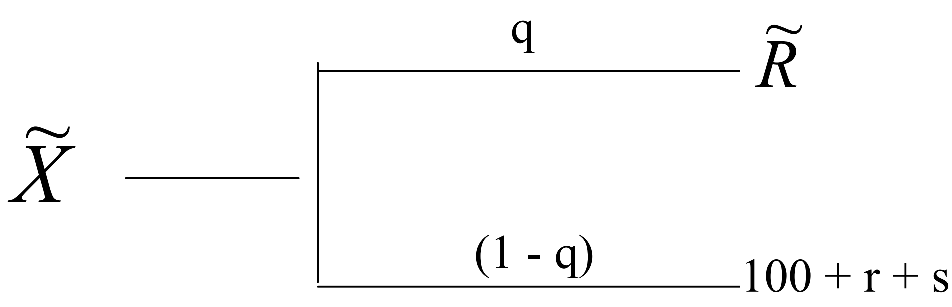
Figure 1: Insured Catastrophe Losses, 1949 – 1997 (in 1997 dollars)



Source: Insurance Services Office

Figure 2

Valuing Catastrophe Linked Securities Using a One Period Binomial Model[1]



where:

- $\tilde{X}$ is the stochastic dollar amount received at end of period
- $\tilde{L}$ is the stochastic loss amount experienced by the insurer
- L_0 is the attachment point
- L_1 is the exhaustion point
- q is the probability of exceeding the attachment point, L_0
- $\tilde{R}$ is the stochastic recovery value if $\tilde{L} > L_0$
- r is risk free interest rate (r = .055)
- s is the promised spread (s = .04)

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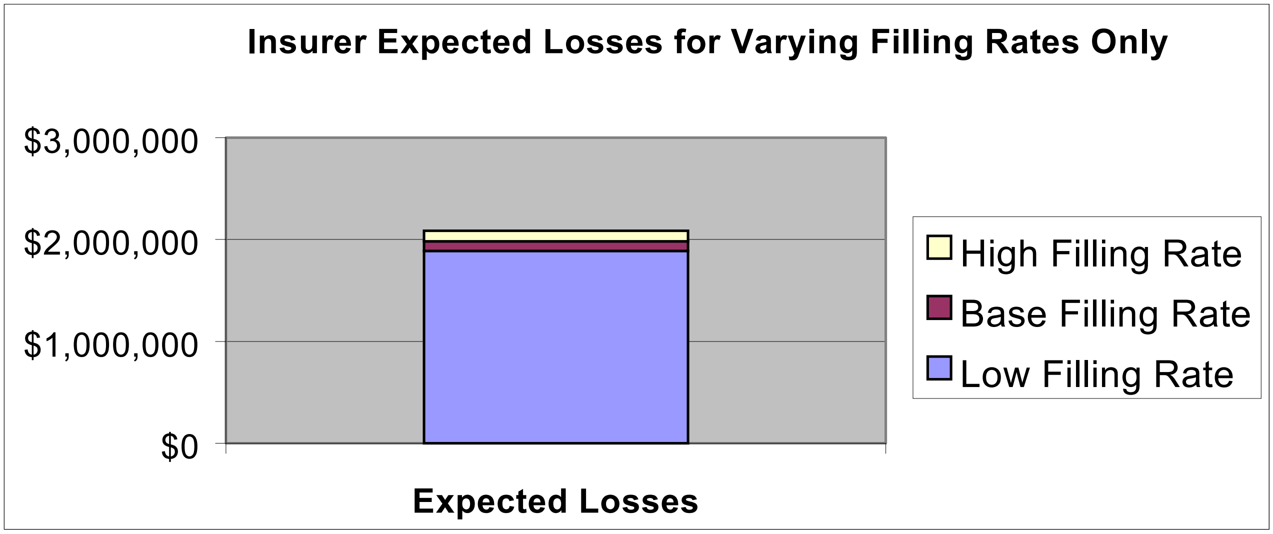
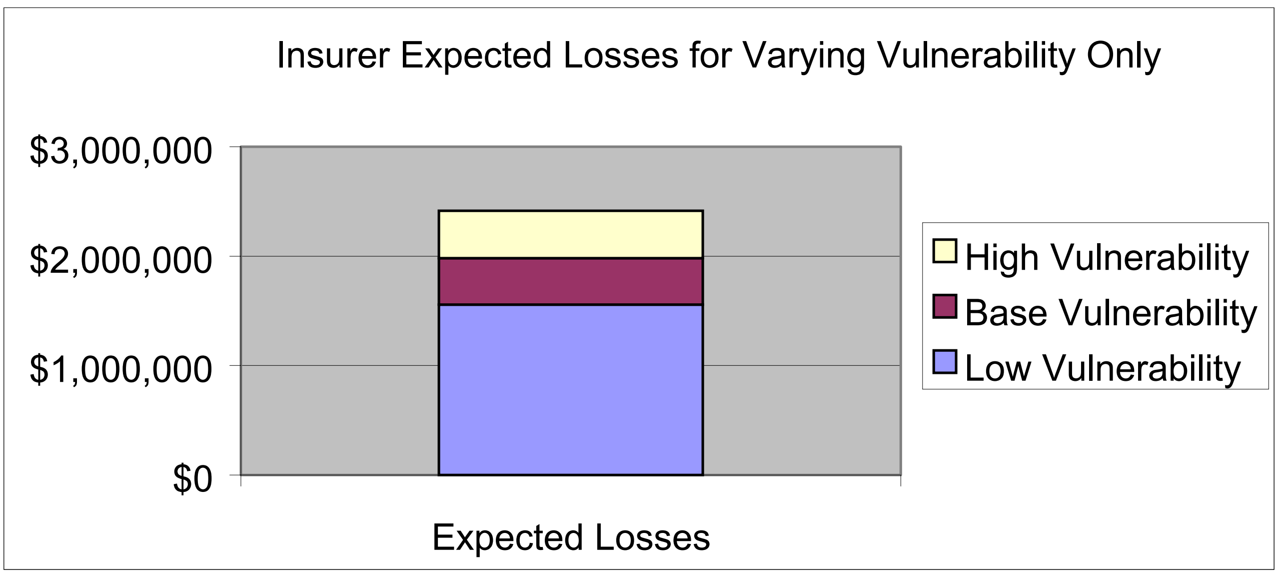
[1] "Analyzing Insurance-Linked Securities", Eduardo Canabarro, Markus Finkemeier, Richard Anderson, and Fouad Bendimerad (Goldman Sachs Fixed Income Research 10/98)

Figure 5: Impact of Uncertainty on Insurer Expected Losses for a Full Book of Business





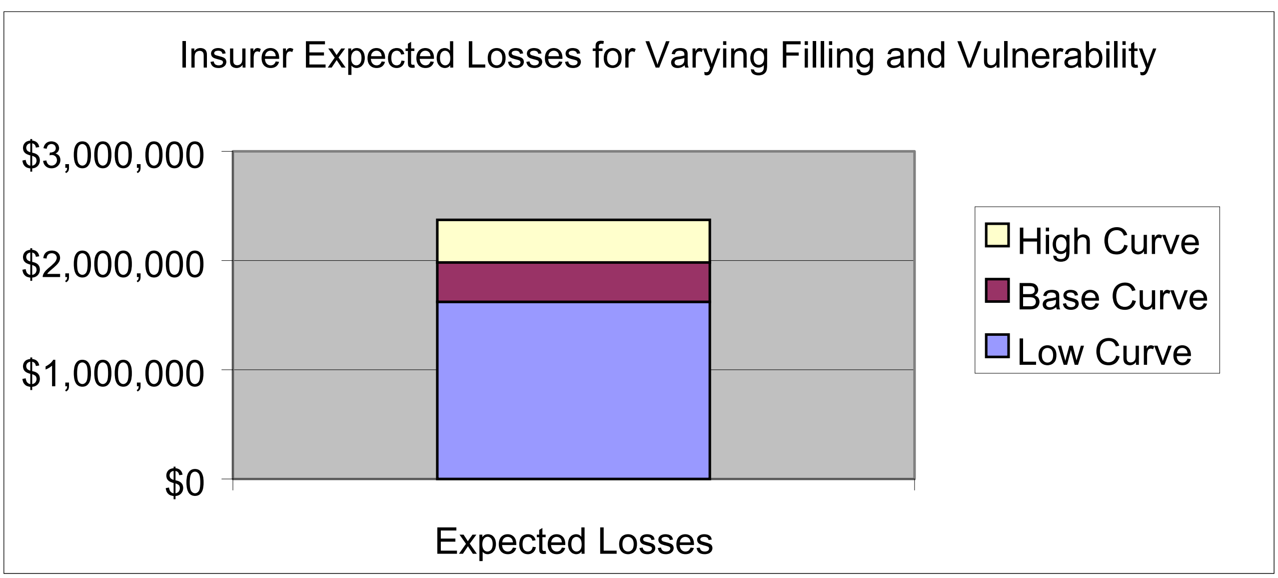


Table 1: Terms of a Hypothetical Cat Bond

Principal	\$36,000,000
Attachment Point (L_0)	\$42,496,000
Attachment Probability	(1.00%)
Exhaustion Point (L_1)	\$82,496,000
Exhaustion Probability	(0.21%)
Layer	\$40,000,000
Coinsurance	10%
Spread (s)	4.00%
Risk Free Rate (r)	5.50%
Coupon	\$3,420,000
Payment if $L < L_0$	\$39,420,000

Table 2: Relative Value Analysis*

Speculative Grade	Historical (1983-97) Default Probabilities	Spread Over LIBOR**	Recovery Rate [%]	Std Dev of Recovery	Std Dev of Return	Expected Loss	Sharpe Ratio
	p		E(R)	SD(R)	SD(V)		
Ba2	0.60%	1.10%	51.26	25.81	4.75	0.33%	0.25
Ba3	2.70%	1.36%	51.26	25.81	10.02	1.51%	0.02
B1	3.80%	1.84%	51.26	25.81	11.91	2.15%	0.01
B2	6.70%	2.00%	51.26	25.81	15.66	3.79%	-0.09
B3	13.20%	2.49%	51.26	25.81	21.49%	7.54%	-0.22
Principal at Risk Tranche	Attachment Probs						
Res Re '97	1.00%	5.82%	48.30	30.60	7.01	0.63%	0.80
Parametric	1.02%	4.36%	41.23	30.04	7.57	0.70%	0.54
Trinity	1.53%	3.67%	54.61	38.27	8.14	0.83%	0.39
Res Re '98	0.87%	4.04%	42.67	35.72	7.06	0.58%	0.54
Mosaic Class A	1.13%	4.40%	61.40	30.05	6.06	0.55%	0.70
Mosaic Class B	4.29%	8.20%	52.98	32.71	14.09	2.62%	0.42
Principal Protected Tranche	Attachment Probs						
Res Re '97	1.00%	2.76%	75.05	16.22	3.72	0.34%	0.76
Parametric	1.02%	2.09%	73.47	15.02	3.78	0.35%	0.56
Trinity	1.53%	1.57%	80.91	18.14	3.86	0.39%	0.39
Mosaic	1.13%	2.15%	83.53	15.03	3.03	0.28%	0.75

* For CAT bonds, they multiplied the quoted spreads by #d/360, where #d is the total number of days over which interest is paid

** The authors note that spreads have widened considerably since the summer of 1998. They estimate the new spreads to be 2.70%, 3.00%, 3.80%, 4.20%, and 5.60% for the bonds rated Ba2, Ba3, B1, B2, and B3, respectively. This implies Sharpe ratios of .60, .19, .18, .05, and -.08. We use the new spreads for the analysis in this paper.

Source: Canabarro et al (1998)

Table 3: Composition of Book of Business

	# Properties
Wood Frame	496
Masonry Veneer	1,005
Masonry	3,117
Semi-Wind Resistive	260
Wind Resistive	122
Total	5,000

Table 4: The Impact of Uncertainty in the Right Tail

Probability of Exceeding:

	L_0 = 42,496,000	L_1 = 82,496,000
Curve:		
Low	0.71%	0.10%
Base	1.00%	0.21%
High	1.34%	0.36%

Table 5: The Impact of Uncertainty on the Value of a Cat Bond

	Low	**Base**	**High**
E{X}	$39,451,364	$39,375,243	$39,289,701
VAR {X}	2.67313E+12	5.02978E+12	7.93481E+12
STD DEV {X}	1,634,971	2,242,716	2,816,879
Expected return	9.59%	9.38%	9.14%
Excess Return	$1,471,364	$1,395,243	$1,309,701
Sharpe Ratio	0.90	0.62	0.46

Notes:
We define the Sharpe Ratio as (Excess Return)/(STD DEV {X})

We assume that the risk free rate = 5.5% and LIBOR = 5.9%

Our Sharpe Ratios are defined in terms of the risk free rate, not LIBOR.

Table 6: Sharpe Ratios vs. Spread

Spread (%)	Low	Medium	High
1.50	0.35	0.22	0.15
1.60	0.37	0.24	0.16
1.70	0.39	0.25	0.17
1.75	0.40	0.26	0.18
1.80	0.42	0.27	0.18
1.90	0.44	0.29	0.20
2.00	0.46	0.30	0.21
2.10	0.48	0.32	0.22
2.20	0.50	0.33	0.23
2.25	0.51	0.34	0.24
2.30	0.53	0.35	0.25
2.40	0.55	0.37	0.26
2.50	0.57	0.38	0.27
2.60	0.59	0.40	0.29
2.70	0.61	0.41	0.30
2.75	0.62	0.42	0.31
2.80	0.64	0.43	0.31
2.90	0.66	0.45	0.32
3.00	0.68	0.46	0.34
3.10	0.70	0.48	0.35
3.20	0.72	0.49	0.36
3.25	0.73	0.50	0.37
3.30	0.75	0.51	0.38
3.40	0.77	0.53	0.39
3.50	0.79	0.54	0.40
3.60	0.81	0.56	0.41
3.70	0.83	0.57	0.43
3.75	0.84	0.58	0.43
3.80	0.86	0.59	0.44
3.90	0.88	0.61	0.45
4.00	0.90	0.62	0.46
4.10	0.92	0.64	0.48
4.20	0.94	0.65	0.49
4.25	0.95	0.66	0.50
4.30	0.97	0.67	0.50
4.40	0.99	0.69	0.52
4.50	1.01	0.70	0.53

Table 7: Rate of Return vs. Spread

Spread (%)	Low	Medium	High
1.50	7.09%	6.88%	6.64%
1.60	7.19%	6.98%	6.74%
1.70	7.29%	7.08%	6.84%
1.75	7.34%	7.13%	6.89%
1.80	7.39%	7.18%	6.94%
1.90	7.49%	7.28%	7.04%
2.00	7.59%	7.38%	7.14%
2.10	7.69%	7.48%	7.24%
2.20	7.79%	7.58%	7.34%
2.25	7.84%	7.63%	7.39%
2.30	7.89%	7.68%	7.44%
2.40	7.99%	7.78%	7.54%
2.50	8.09%	7.88%	7.64%
2.60	8.19%	7.98%	7.74%
2.70	8.29%	8.08%	7.84%
2.75	8.34%	8.13%	7.89%
2.80	8.39%	8.18%	7.94%
2.90	8.49%	8.28%	8.04%
3.00	8.59%	8.38%	8.14%
3.10	8.69%	8.48%	8.24%
3.20	8.79%	8.58%	8.34%
3.25	8.84%	8.63%	8.39%
3.30	8.89%	8.68%	8.44%
3.40	8.99%	8.78%	8.54%
3.50	9.09%	8.88%	8.64%
3.60	9.19%	8.98%	8.74%
3.70	9.29%	9.08%	8.84%
3.75	9.34%	9.13%	8.89%
3.80	9.39%	9.18%	8.94%
3.90	9.49%	9.28%	9.04%
4.00	9.59%	9.38%	9.14%
4.10	9.69%	9.48%	9.24%
4.20	9.79%	9.58%	9.34%
4.25	9.84%	9.63%	9.39%
4.30	9.89%	9.68%	9.44%
4.40	9.99%	9.78%	9.54%
4.50	10.09%	9.88%	9.64%

Table 8: Utility Based Prices as a Function of Investor Risk Aversion

	Base Curve		High Curve			Low Curve	
Risk Aversion	Required Spread	Risk Premia	Required Spread	Risk Premia	Uncertainty Premia	Required Spread	Risk Premia
0	0.52%	-	0.76%	-	0.0%	0.31%	-
0.5	0.53%	0.01%	0.78%	0.01%	0.24%	0.32%	0.01%
1 (log)	0.54%	0.02%	0.79%	0.03%	0.24%	0.32%	0.01%
2	0.56%	0.04%	0.83%	0.06%	0.24%	0.33%	0.02%
5	0.63%	0.11%	0.93%	0.17%	0.24%	0.37%	0.06%
10	0.78%	0.25%	1.17%	0.40%	0.24%	0.44%	0.13%
15	0.97%	0.45%	1.49%	0.72%	0.24%	0.54%	0.23%
20	1.24%	0.71%	1.92%	1.16%	0.24%	0.68%	0.36%
25	1.60%	1.08%	2.53%	1.76%	0.24%	0.86%	0.55%
30	2.11%	1.58%	3.36%	2.60%	0.24%	1.12%	0.81%
35	2.80%	2.28%	4.50%	3.74%	0.24%	1.47%	1.16%
40	3.76%	3.23%	6.03%	5.26%	0.24%	1.97%	1.66%
45	5.05%	4.52%	8.02%	7.25%	0.24%	2.67%	2.36%

Notes:
We assume that the representative agent's preferences can be described by a power utility function with constant relative risk aversion.
Required spreads are solved for using the method described in the text. We assume the agent invests 10% of his wealth in cat bonds.

Table 9: Utility Based Prices for High Yield Bonds as a Function of Investor Risk Aversion

	Ba2		Ba3		B1		B2		B3
Risk Aversion	Required Spread	Risk Aversion	Required Spread	Risk Aversion	Required Spread	Risk Aversion	Required Spread	Risk Aversion	Required Spread
0	0.33%	0	1.50%	0	2.14%	0	3.89%	0	8.24%
0.5	0.33%	0.5	1.53%	0.5	2.18%	0.5	3.96%	0.5	8.40%
1 (log)	0.34%	1 (log)	1.56%	1 (log)	2.21%	1 (log)	4.03%	1 (log)	8.55%
2	0.35%	2	1.61%	2	2.29%	2	4.17%	2	8.88%
5	0.39%	5	1.79%	5	2.55%	5	4.66%	5	9.99%
10	0.47%	10	2.16%	10	3.09%	10	5.68%	10	12.41%
15	0.57%	15	2.65%	15	3.81%	15	7.08%	15	15.93%
20	0.71%	20	3.32%	20	4.80%	20	9.07%	20	21.45%
25	0.89%	25	4.26%	25	6.20%	25	12.04%	25	31.35%
30	1.14%	30	5.60%	30	8.27%	30	16.83%	30	56.44%
35	1.50%	35	7.60%	35	11.50%	35	25.81%	35	------
40	2.01%	40	10.79%	40	17.14%	40	52.25%	40	------
45	2.74%	45	16.54%	45	29.70%	45	------	45	------
Low Spread	1.10%		1.36%		1.84%		2.00%		2.49%
High Spread	2.70%		3.00%		3.80%		4.20%		5.60%

Table 10: Fixed Cost of Education Necessary to Leave Investor Indifferent Between Cat Bond and High Yield Bonds

Risk Aversion	Ba2	Ba3	B1	B2	B3
0	$ 403,143	$ 731,262	$ 682,860	$ 1,157,441	$2,097,160
0.5	$ 401,569	$ 737,126	$ 692,598	$ 1,177,435	$2,138,033
1	$ 400,665	$ 743,462	$ 702,570	$ 1,197,831	$2,179,762
2	$ 398,161	$ 755,453	$ 723,097	$ 1,239,120	$2,265,856
5	$ 387,348	$ 796,183	$ 791,353	$ 1,374,968	$ 2,546,331
10	$ 365,655	$ 877,768	$ 929,106	$ 1,646,954	$ 3,095,574
15	$ 335,329	$ 980,905	$ 1,103,727	$ 1,987,448	$ 3,759,526
20	$ 292,572	$ 1,111,203	$ 1,324,701	$ 2,410,332	$ 4,546,675
25	$ 231,993	$ 1,274,883	$ 1,602,037	$ 2,927,077	$ 5,452,691
30	$ 146,174	$ 1,477,690	$ 1,944,048	$ 3,541,373	$ 6,453,118
35	$ 25,430	$ 1,722,662	$ 2,353,177	$ 4,241,683	$ 7,498,693
40	$ (141,757)	$ 2,006,521	$ 2,819,906	$ 4,993,930	$ 8,516,880
45	$ (366,815)	$ 2,315,235	$ 3,316,711	$ 5,739,030	$ 9,421,660

Table 11: Investor Preferences Under a Rank Dependent Expected Utility Function

	RDEU	Fixed Cost *
Risk Free	347588	
Cat Bond	327791	
Ba2	321256	$ 43,500
Ba3	13198	$ 1,985,500
B1	-74993	$ 2,505,950
B2	-331327	$ 3,654,500
B3	-744344	$ 4,679,500

Notes:
*The Fixed Costs column refers to the fixed cost of education necessary to leave the investor indifferent between the cat bond and the high yield bond.

Table 12

Ratios* of Underwriters Premiums for Ambiguous and/or Uncertain Earthquake Risks Relative to Well-Specified Risks

SCENARIO	CASES				
	1	2	3	4	
	p,L	Ap,L	p,UL	Ap,UL	N
p=.005 L=$1 million	1	1.28	1.19	1.77	17
p=.005 L=$10 million	1	1.31	1.29	1.59	8
p=.01 L=$1 million	1	1.19	1.21	1.50	23
p=.01 L=$10 million	1	1.38	1.15	1.43	6

N= Number of Respondents

* Ratios are based on Mean Premiums Across Number of Respondents for Each Scenario

Source: Adapted from Table 3 in Kunreuther et. al. (1995)

Table 13: Required Spreads Based on Empirical Evidence on Ambiguity Aversion

	Ap,L	p,UL	Ap,UL
Ratios	1.38	1.15	1.43
Ba2	5.49%	3.59%	5.91%
Ba3	4.24%	2.55%	4.60%
B1	4.42%	2.70%	4.80%
B2	2.60%	1.185%	2.91%
B3	-1.00%	-1.82%	0.82%

Note: Ratios are for the p=.01, L=10 mil Scenario shown in Table 12.

Figure 3: Base Case EP Curve Without Uncertainty

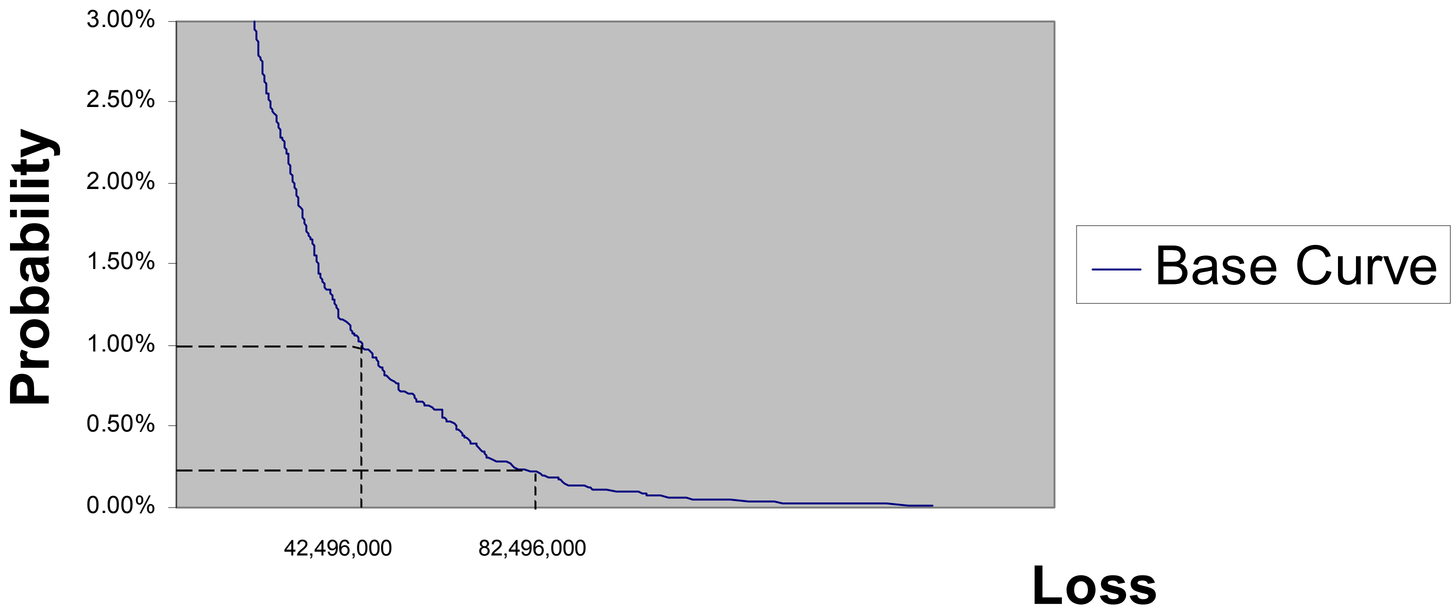


Figure 4: The Impact of Uncertainty